TMD's at High Density: Quasi-Classical Approximation

Part 1 of 2

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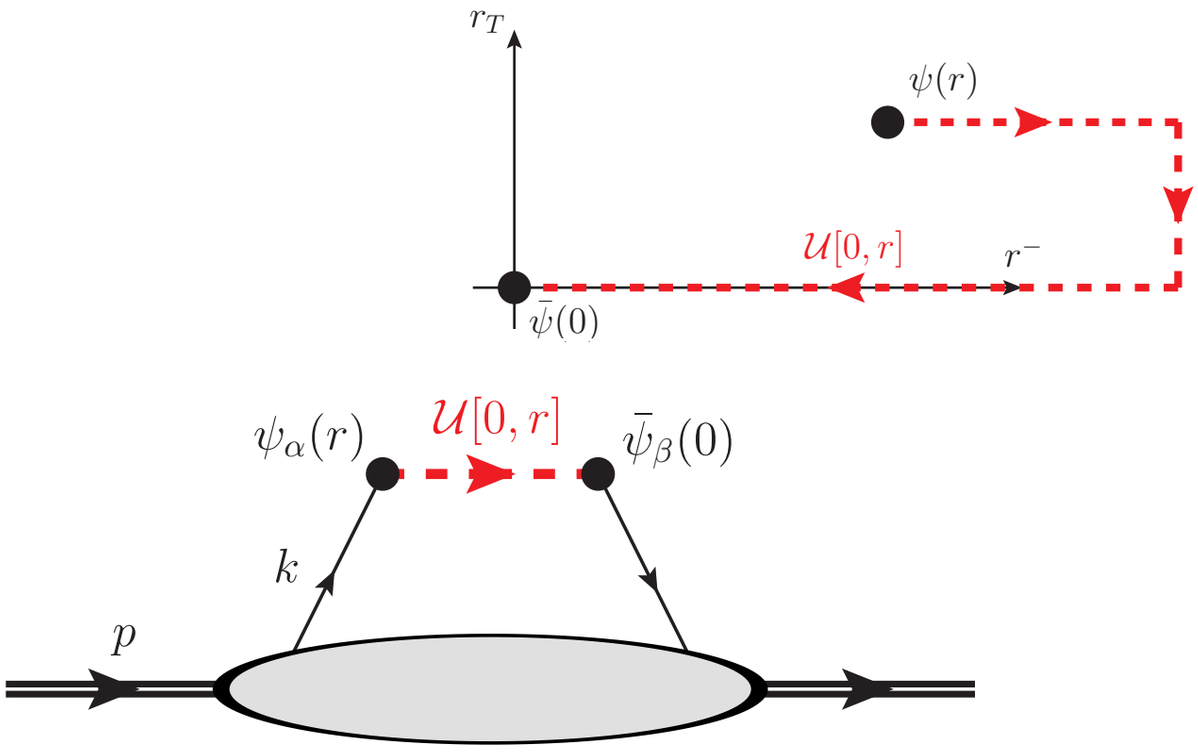
EIC Users' Group Meeting

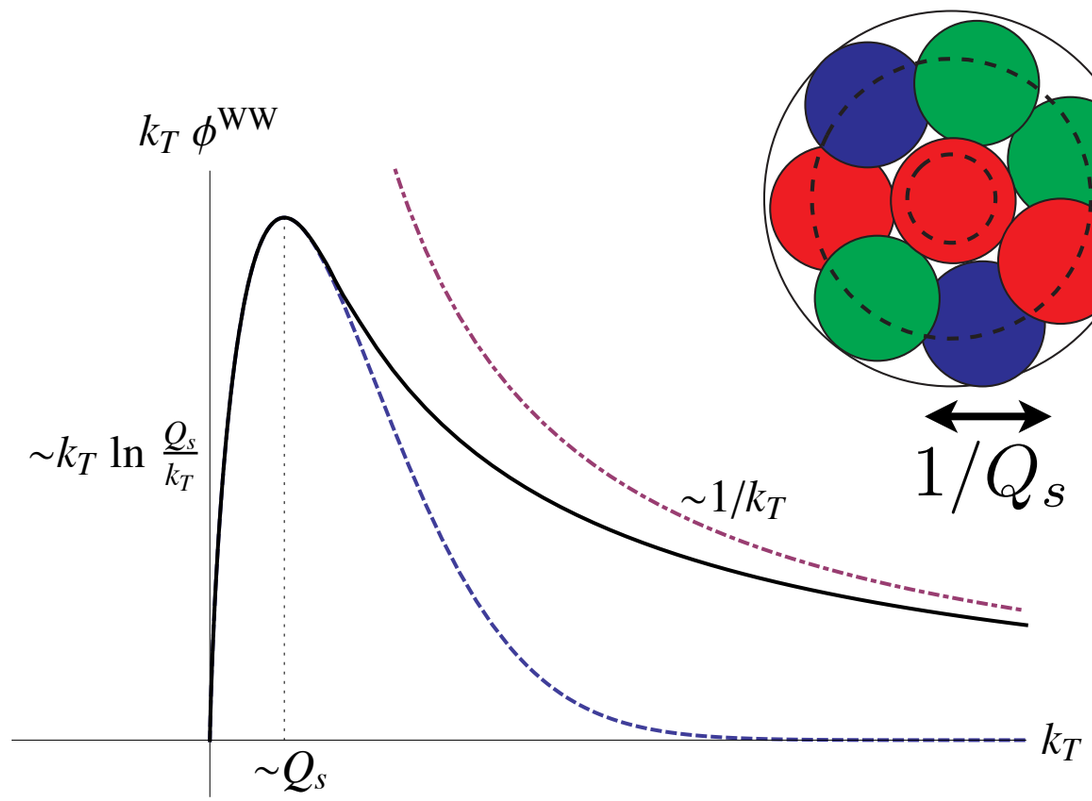
Friday Jan. 8, 2016

Overview

TMD's: Quark Spin / Orbit Distributions



High-Density Gluon Fields: Intrinsic Hard Scale Q_s



- Hard scale Q_s leads to factorization of the TMD's themselves.
- Calculable structure from first-principles QCD

Quark Anatomy of a Spin-1/2 Hadron

$$\phi_{\alpha\beta}(x, \vec{k}_\perp) = \int \frac{d^{2-}r}{(2\pi)^3}\, e^{ik\cdot r} \langle h(p,S)|\, \bar{\psi}_\beta(0)\, \mathcal{U}[0,r]\, \psi_\alpha(r)\, |h(p,S)\rangle$$

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Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 =		$h_1^\perp$ = − Boer-Mulders
L		g_{1L} = − Helicity	$h_{1L}^\perp$ = −
T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = −	h_1 = − Transversity; $h_{1T}^\perp$ = −
Γ	γ^+	$\gamma^+\gamma^5$	$\gamma^+\gamma^i_\perp\gamma^5$

- 3D distributions of quarks with spin and momentum.

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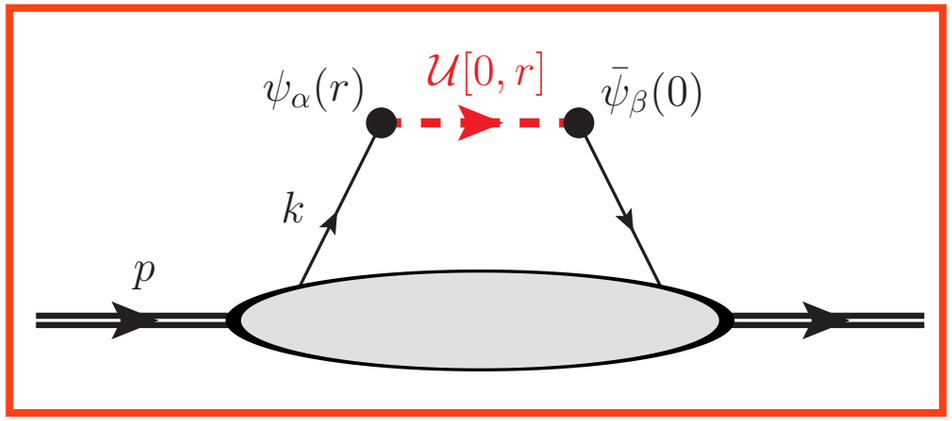

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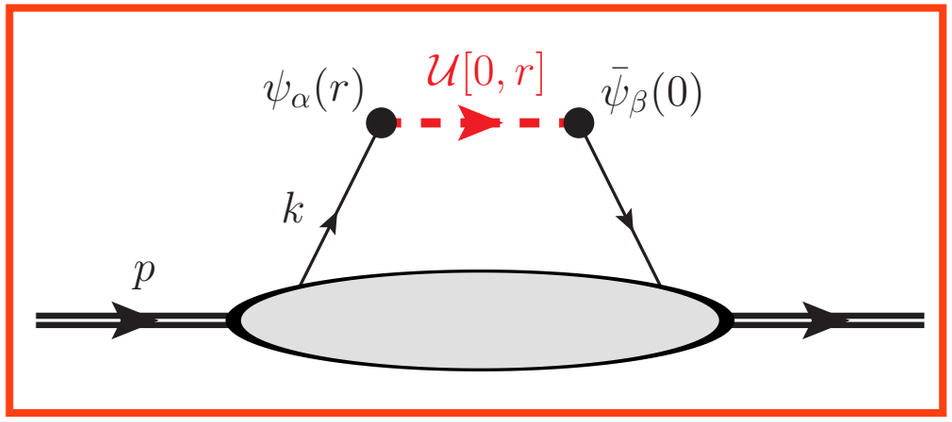

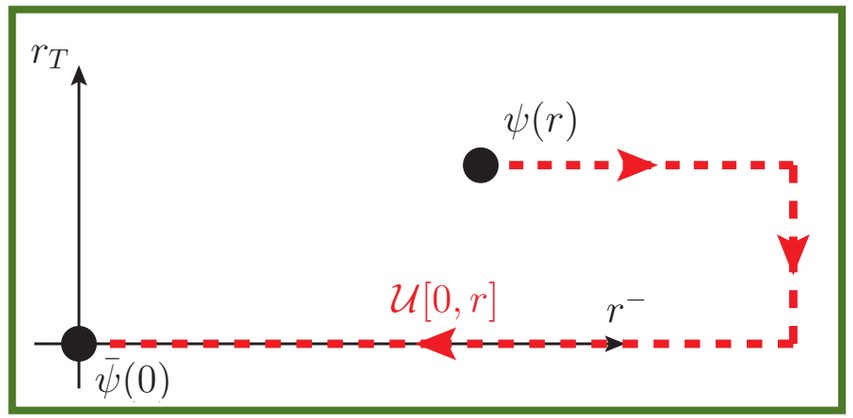

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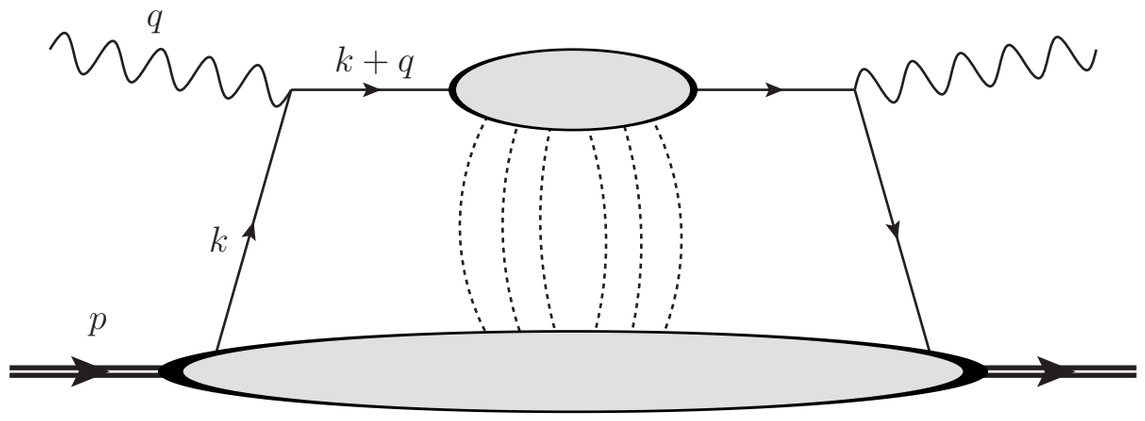
- 3D distributions of quarks with spin and momentum.
- Bilocal quark fields, connected by a staple-shaped gauge link.
- Gauge fields describe physical distortion of quark momentum.

Gauge Fields: Dilute vs. Dense Systems

Dilute Systems: Lensing



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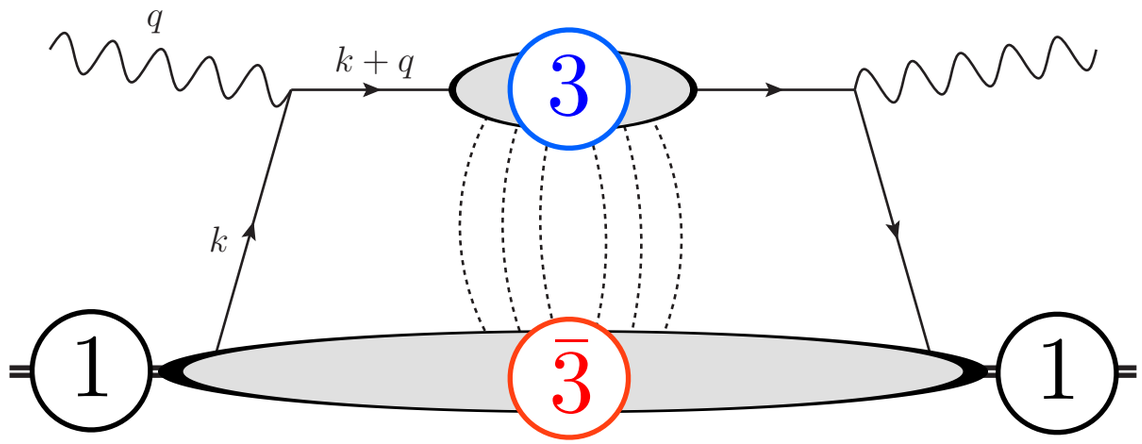

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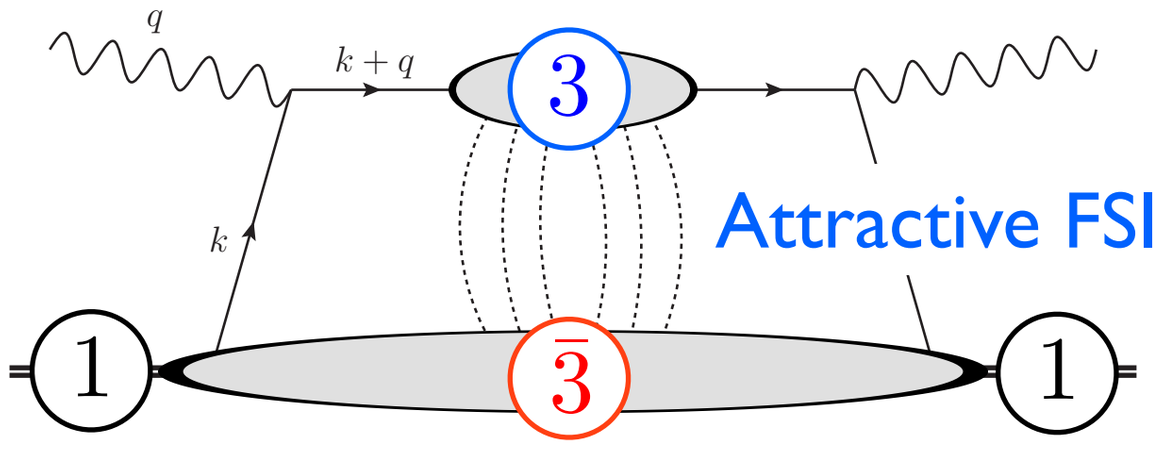

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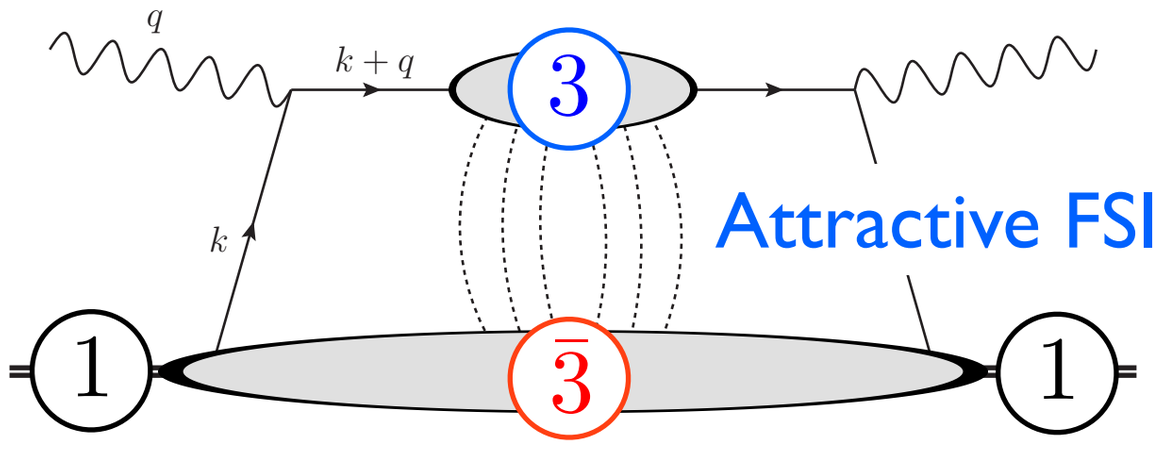

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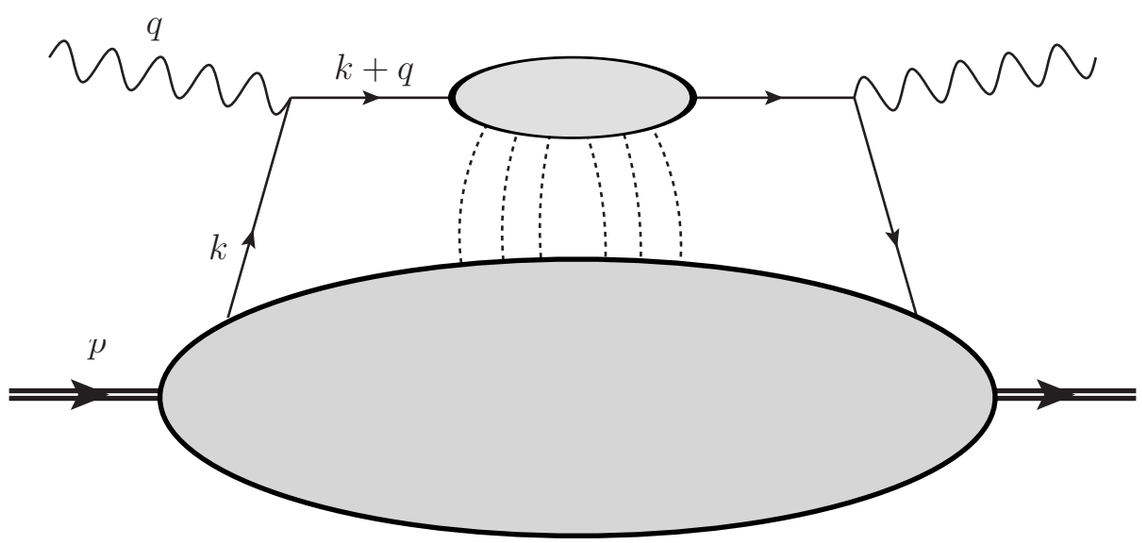

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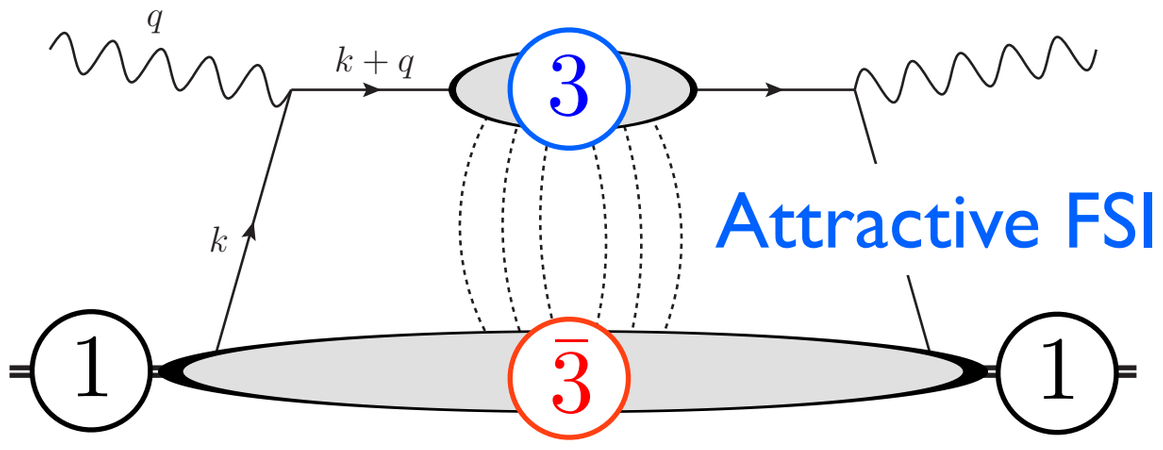

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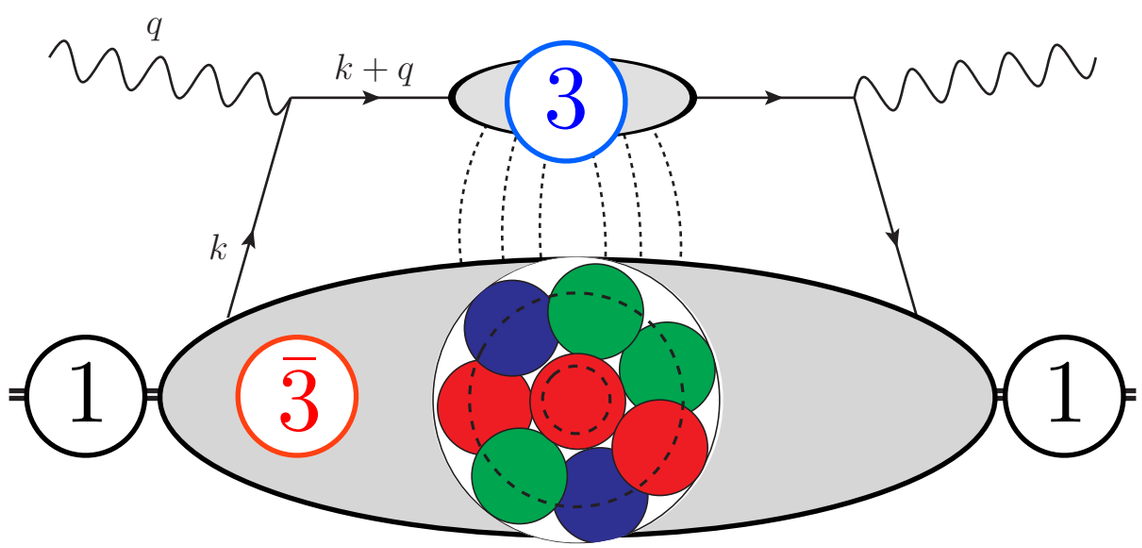

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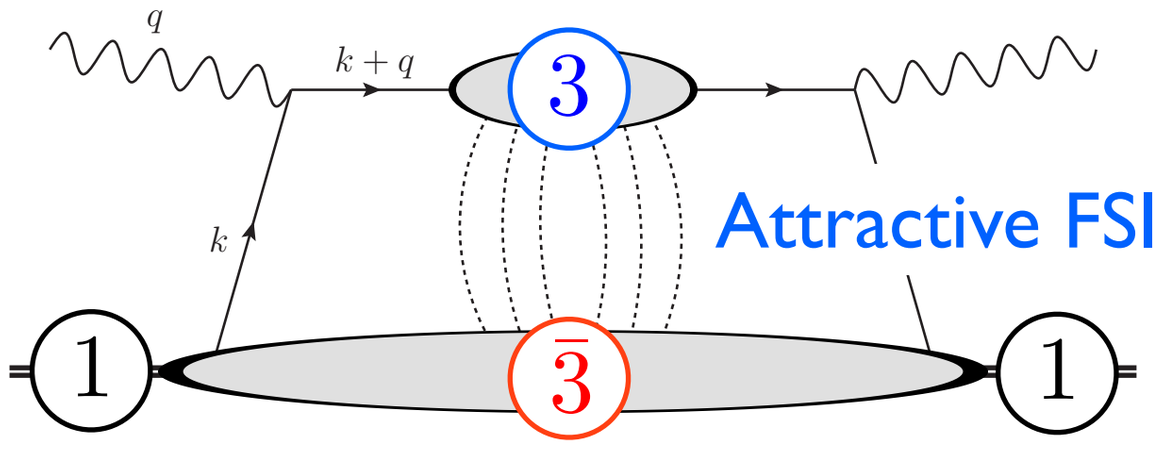

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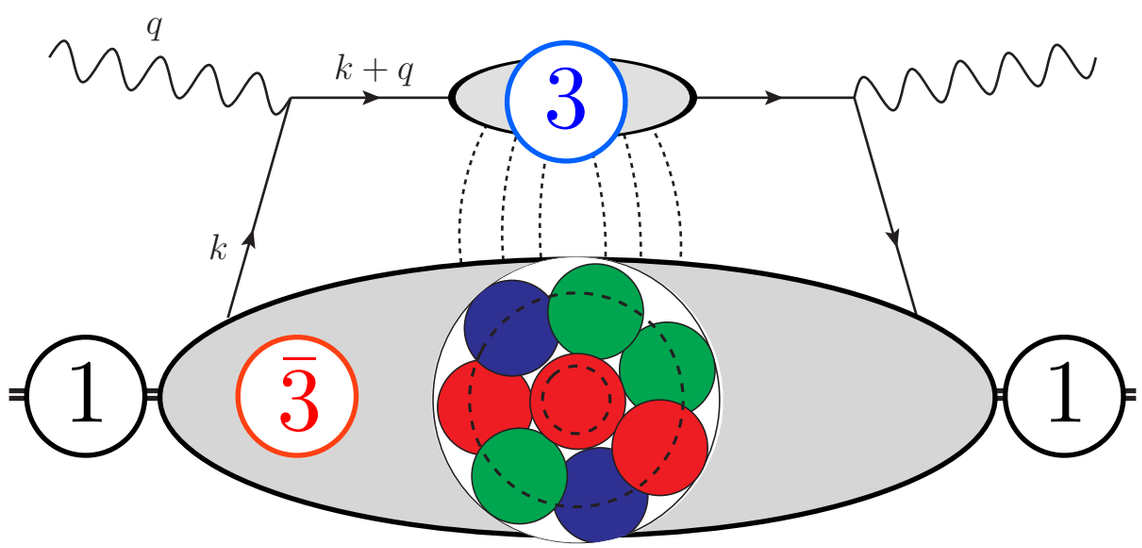

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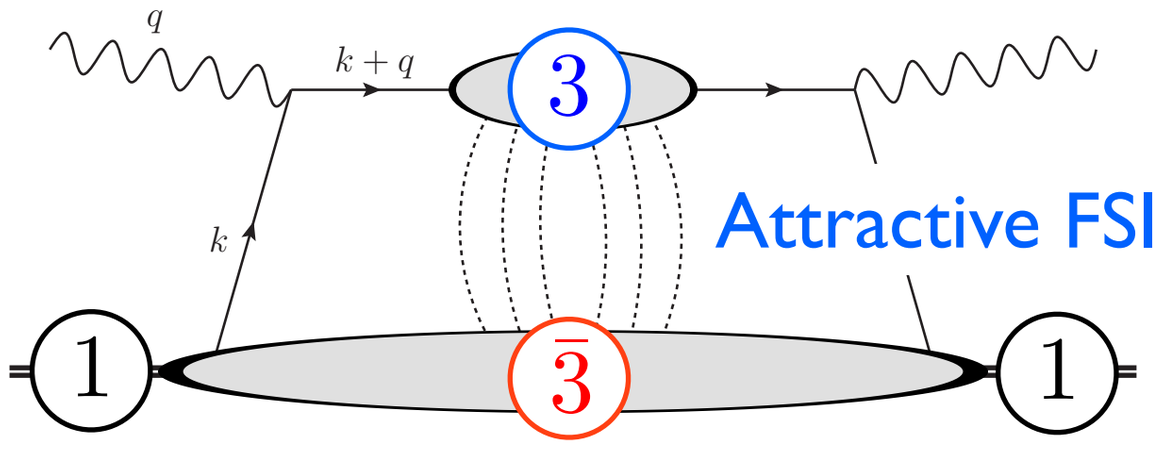

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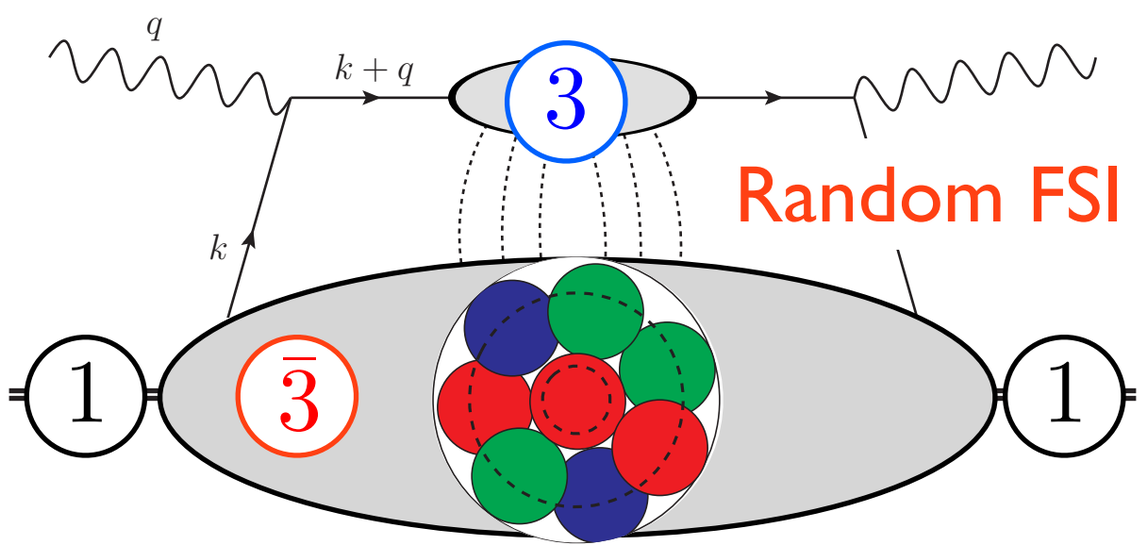
- FSI are random, broadening the quark momentum isotropically.



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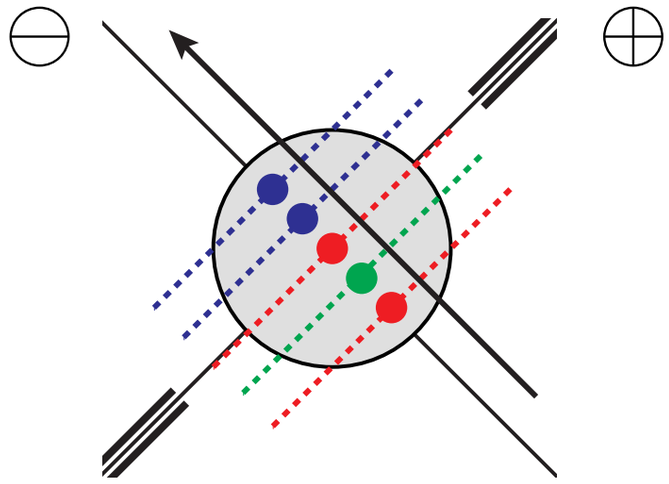
- At high density and weak coupling, multiple independent scattering occurs on the local charge density.

Nucleus: $A \gg 1$ Proton: $\rho \gg 1$



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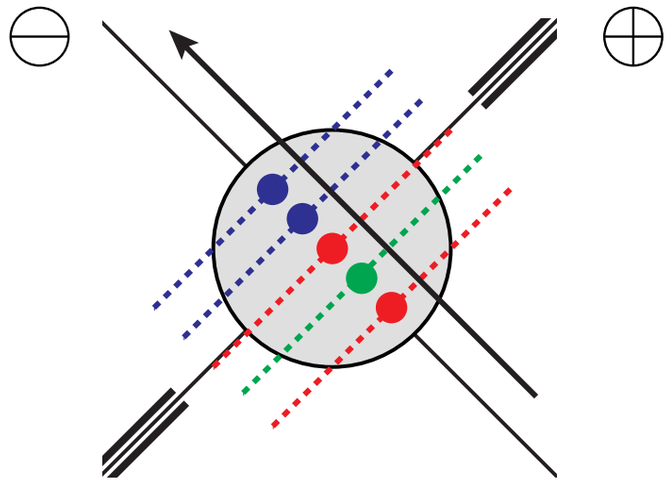
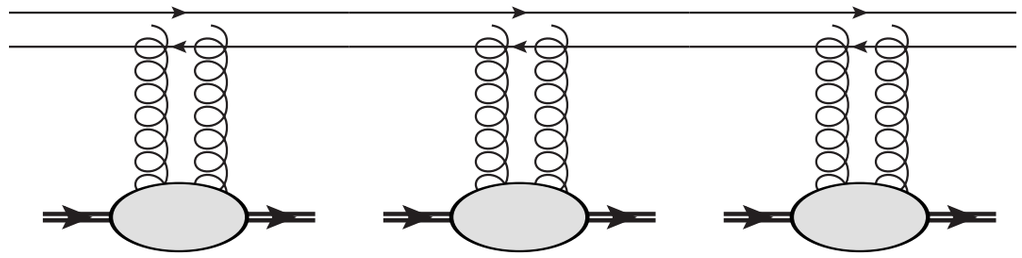


- Resumming the high-density effects leads to scattering in a classical background field.



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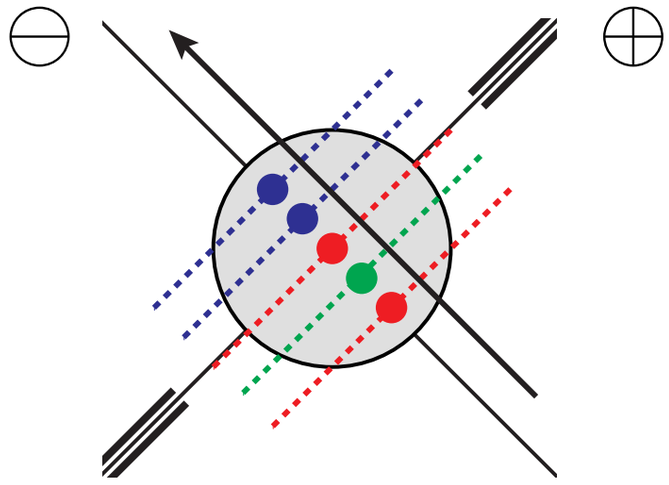

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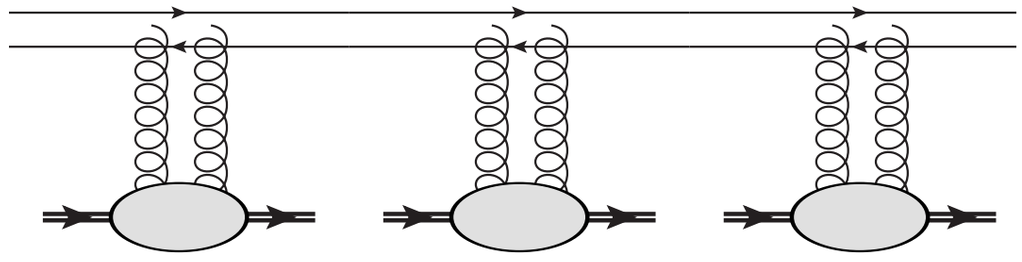

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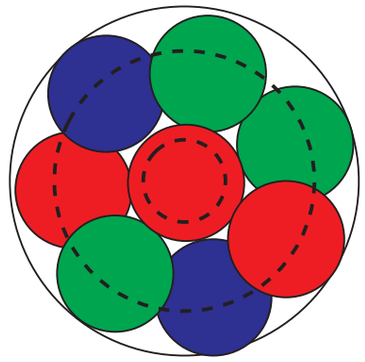
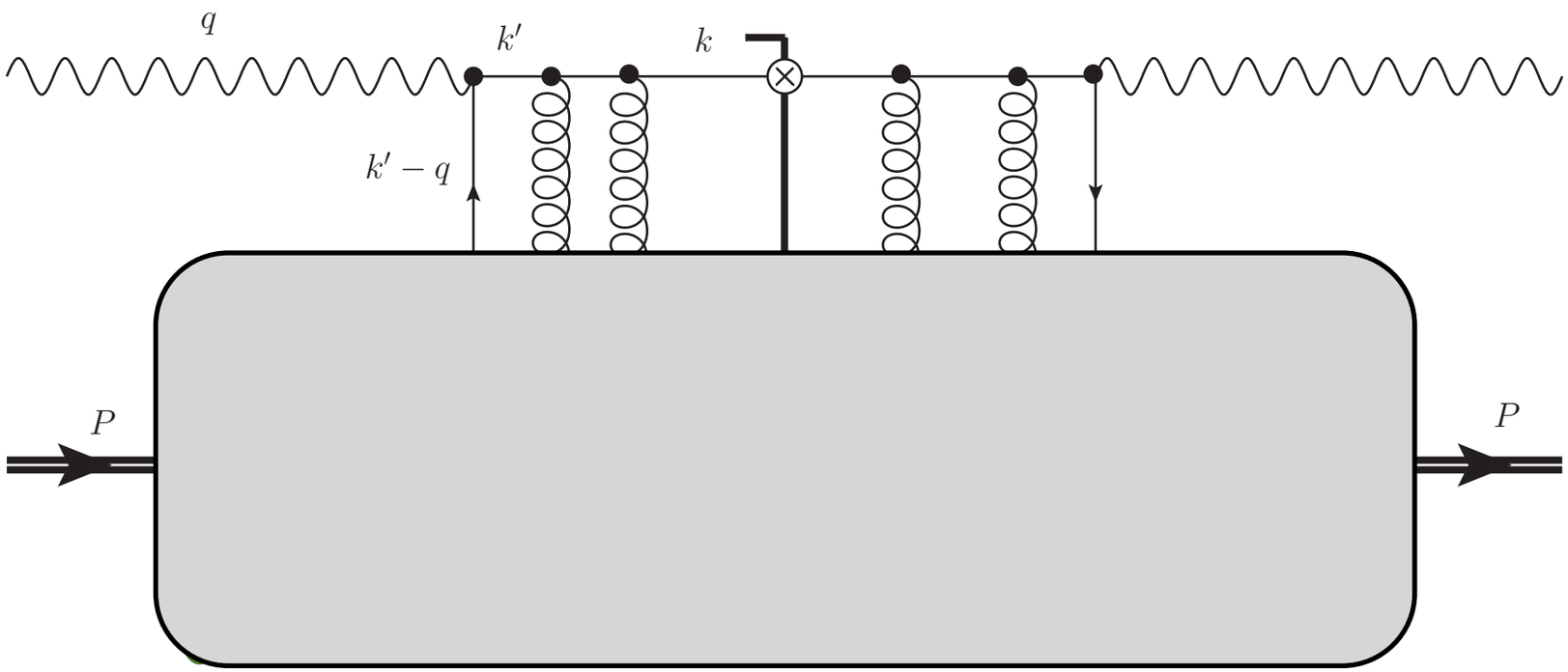
- High charge density defines a hard momentum scale Q_s which dynamically screens the IR gluon field.

Both: $Q_s^2 \propto \alpha_s^2 A^{1/3} \propto \alpha_s \rho$

$Q_s^2 \gg \Lambda^2$



TMD's in the High-Density Limit



$$\langle h(p,S)|\,\bar{\psi}(0)\,\mathcal{U}[0,r]\,\psi(r)\,|h(p,S)\rangle$$

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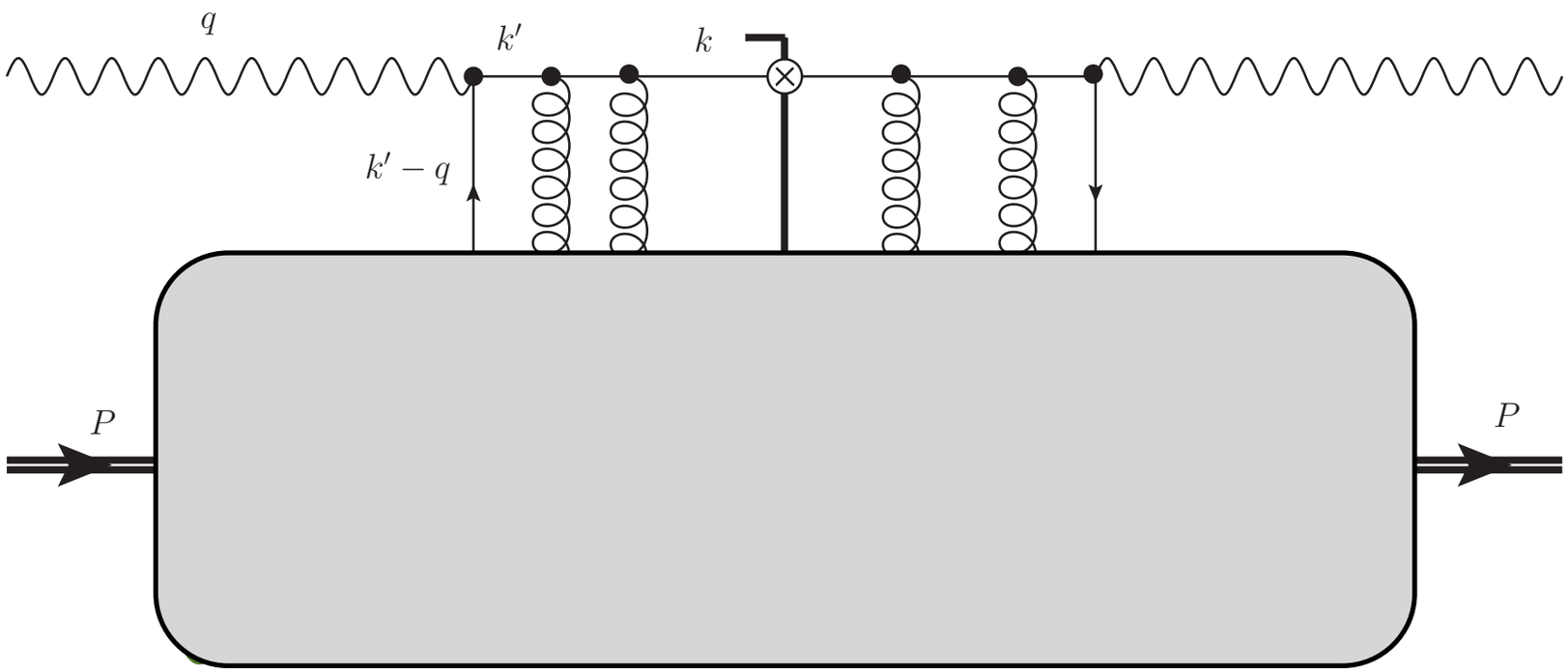

$$\langle A(p,S;Q_s)|\,\bar{\psi}(0)\,\mathcal{U}[0,r]\,\psi(r)\,|A(p,S;Q_s)\rangle$$

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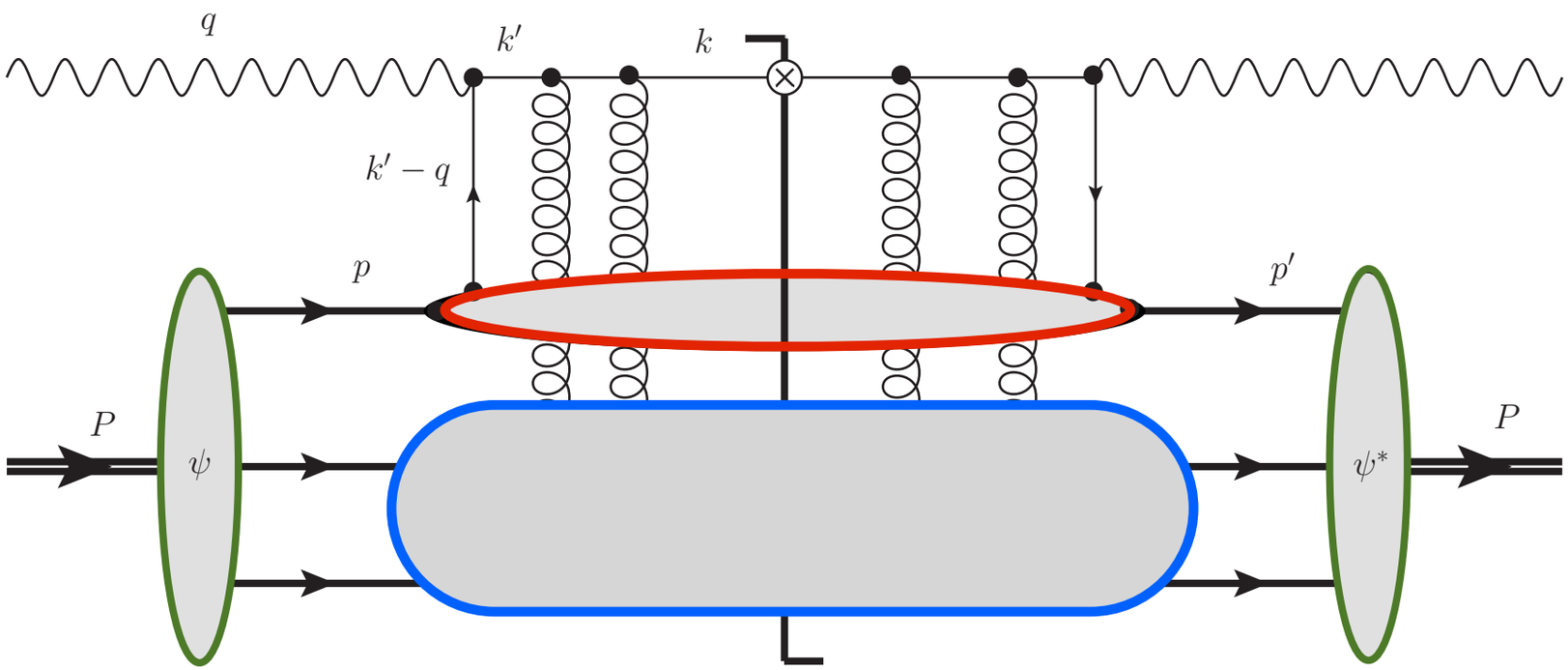

$$\left[\Psi_N^*\,\langle N|\,\bar{\psi}(0)\right]\,\langle(A-1)|\,\mathcal{U}[0,r]\,|(A-1)\rangle\,\left[\psi(r)\,|N\rangle\,\Psi_N\right]$$

Leading order in $A^{1/3}$

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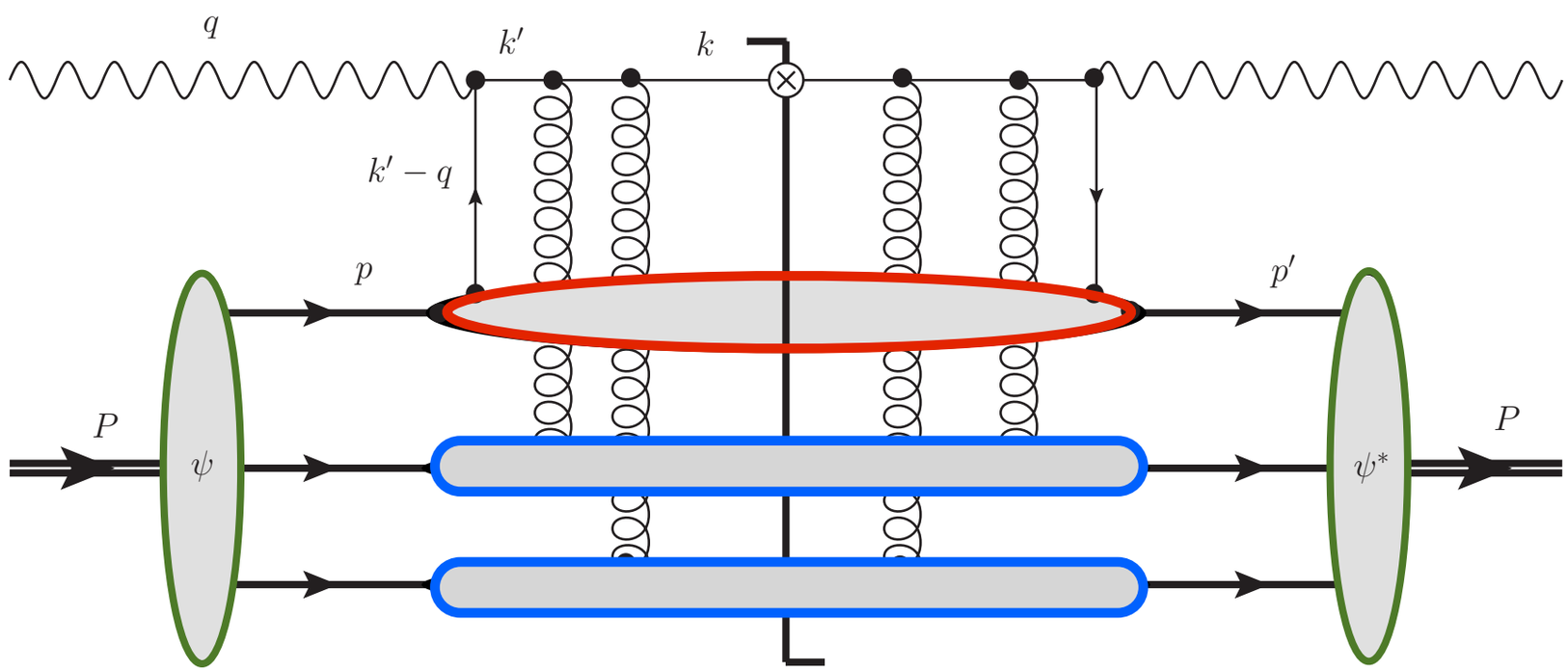

$$\Big[\Psi_N^*\,\langle N|\,\bar{\psi}(0)\Big]\;\underbrace{\langle (A-1)|\;\mathcal{U}[0,r]\;|(A-1)\rangle}_{\alpha_s^2 A^{1/3}\sim\mathcal{O}(1)}\;\Big[\psi(r)\;|N\rangle\;\Psi_N\Big]$$

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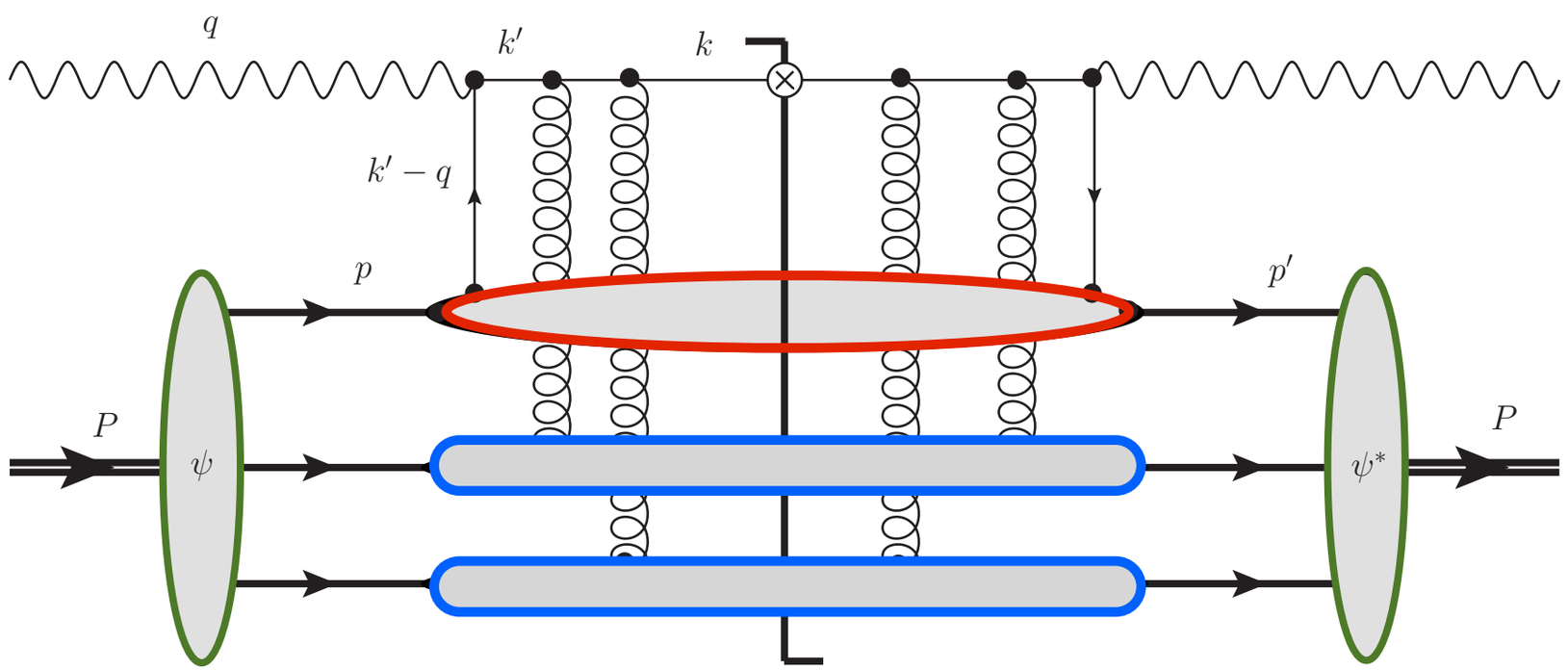

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$$\phi\sim\underbrace{\Big[\Psi_N\Psi_N^*\Big]}_{\text{Nuclear WF}}\;\underbrace{\langle N|\,\bar\psi(0)\,u[0,r]\,\psi(r)\,|N\rangle}_{\text{Nucleonic TMD}}\;\underbrace{\Big[\langle (A-1)|\mathcal{U}[0,r]\,|(A-1)\rangle\Big]}_{\text{Classical Rescattering}}$$

Quasi-Classical Factorization

$$\Phi^A_{\alpha\beta}(x,\vec{k}_\perp) = \frac{A}{(2\pi)^5}\sum_{\sigma\sigma'}\int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\vec{k}_\perp-\vec{k}'_\perp-\hat{x}\vec{p}_\perp)\cdot\vec{r}_\perp}$$

$$\times\; W_{\sigma'\sigma}(p,b)\,[\phi^N_{\alpha\beta}(\hat{x},\vec{k}'_\perp)]_{\sigma\sigma'}\,S^{[\infty^-,b^-]}_{(r_T,b_T)}$$

Nuclear TMD's

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$$\hat{x}\equiv\frac{P^+}{p^+}x$$

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"Average" phase space distribution of nucleons in the nucleus: Leading Order in $A^{1/3}$

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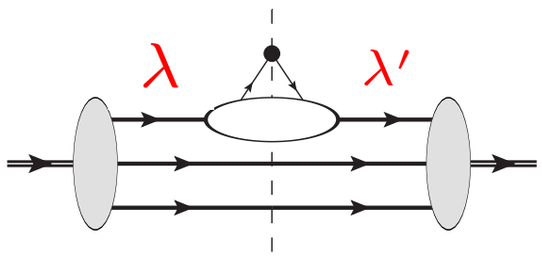
"Average" phase space distribution of nucleons in the nucleus: Leading Order in $A^{1/3}$

$$S^{[\infty^-,b^-]}_{(r_T,b_T)}=\exp\left[-\frac{1}{4}r_T^2Q_s^2(b_T)\left(\frac{R^-(b_T)-b^-}{2R^-(b_T)}\right)\right]$$

Multiple rescattering on spectator nucleons: Resums $\alpha_s^2A^{1/3}\sim\mathcal{O}(1)$

Spin Structure of an Unpolarized Nucleus



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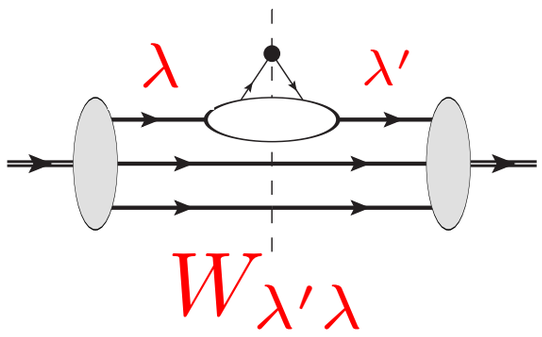

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- The spin of the nucleons is described by the 2 x 2 density matrix $W_{\lambda'\lambda}$

  - In the rest frame: $[W]_{\lambda'\lambda} = W_{unp}[1]_{\lambda'\lambda} + \vec{W}_{pol} \cdot [\vec{\sigma}]_{\lambda'\lambda}$

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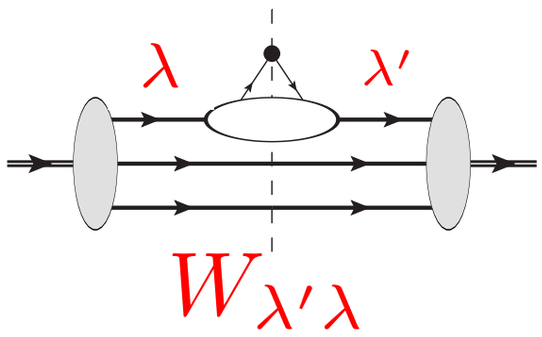

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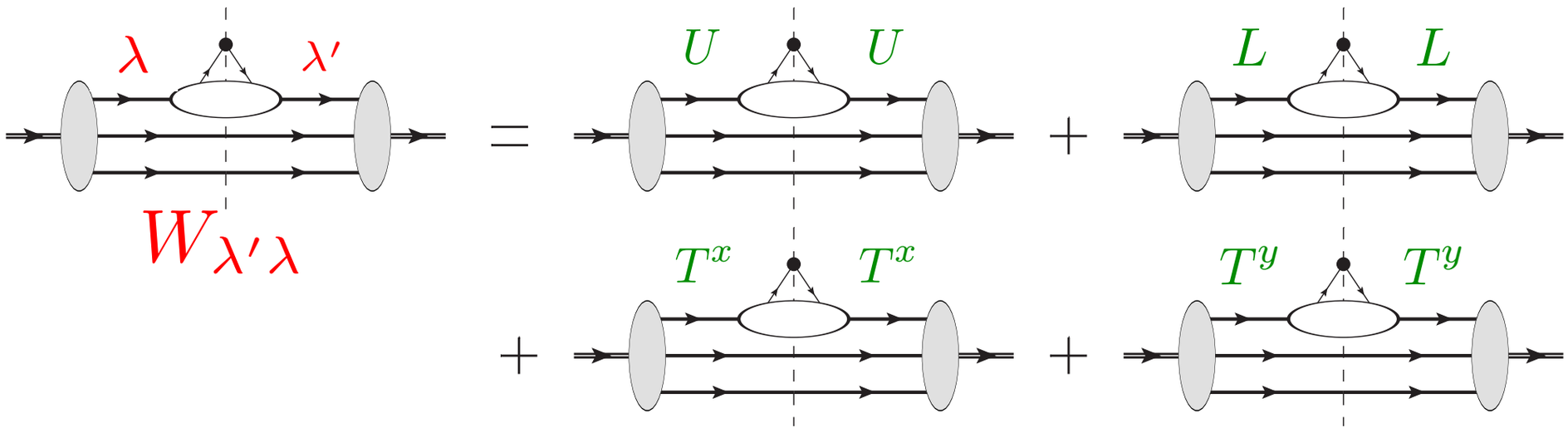

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- The nucleons can have any of 4 polarizations: unpolarized, longitudinal, and transverse (x and y)

Symmetries of the Nuclear Wave Function

$$W_{\sigma'\sigma}(\bar{p}, b) = \frac{1}{2(2\pi)^3} \int \frac{d^{2+}(p-p')}{\sqrt{p^+ p'^+}} e^{-i(p-p')\cdot b} \Psi^N_\sigma(p) \Psi^{N*}_{\sigma'}(p')$$

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  - 3D Rotational Symmetry

?

Proton: Relativistically moving partons....
...Lorentz-Invariance Relations??

D. Pitonyak

Parameterization of the Wigner Distribution

- Imposing P, T, and 3D rotation symmetry:

$$W(\vec{p},\vec{b},\vec{S}) = W_{unp}[\vec{p}^{\,2},\vec{b}^{\,2},(\vec{p}\cdot\vec{b})^2] + \vec{S}\cdot(\vec{b}\times\vec{p})\, W_{OAM}[\vec{p}^{\,2},\vec{b}^{\,2},(\vec{p}\cdot\vec{b})^2]$$

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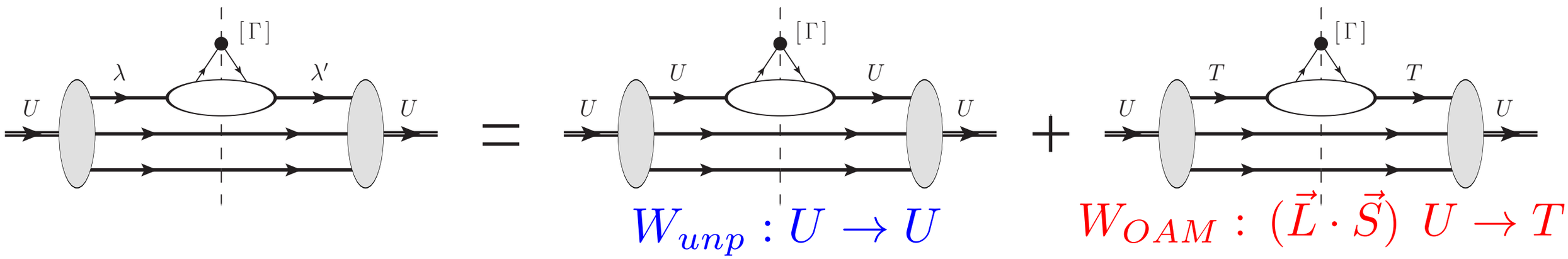
- The maximum spin-orbit structure of the unpolarized nucleus is

$$W(\vec{p},\vec{b},\vec{S}) \Rightarrow W_{unp}[p_T^2,b_T^2;p_z^2,b_z^2] + \underbrace{b_z(\vec{p}_\perp\times\vec{S}_\perp)}_{\text{Depth Dependence!}}\, W_{OAM}[p_T^2,b_T^2;p_z^2,b_z^2]$$

TMD's of an Unpolarized Nucleus



- For an unpolarized nucleus, two channels survive:
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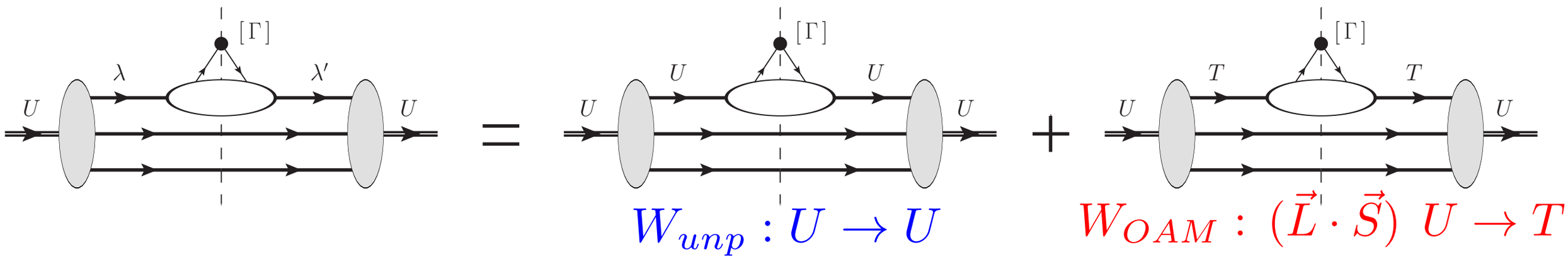

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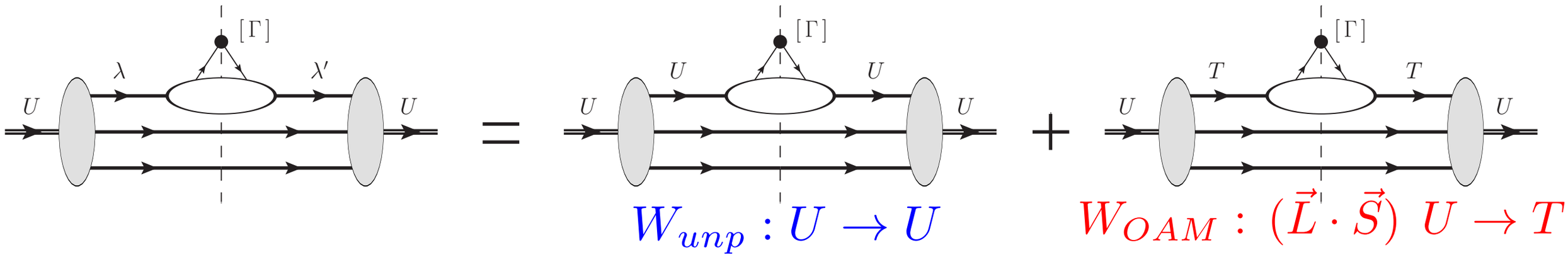

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Unpolarized quarks, azimuthally symmetric

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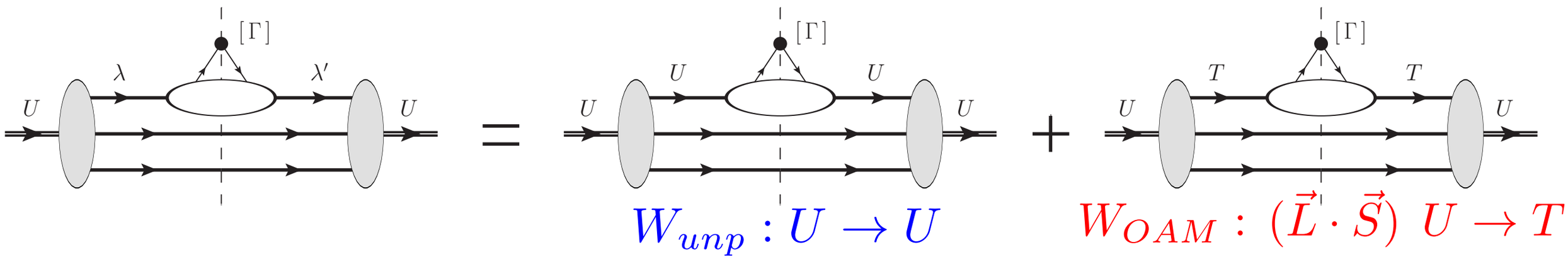

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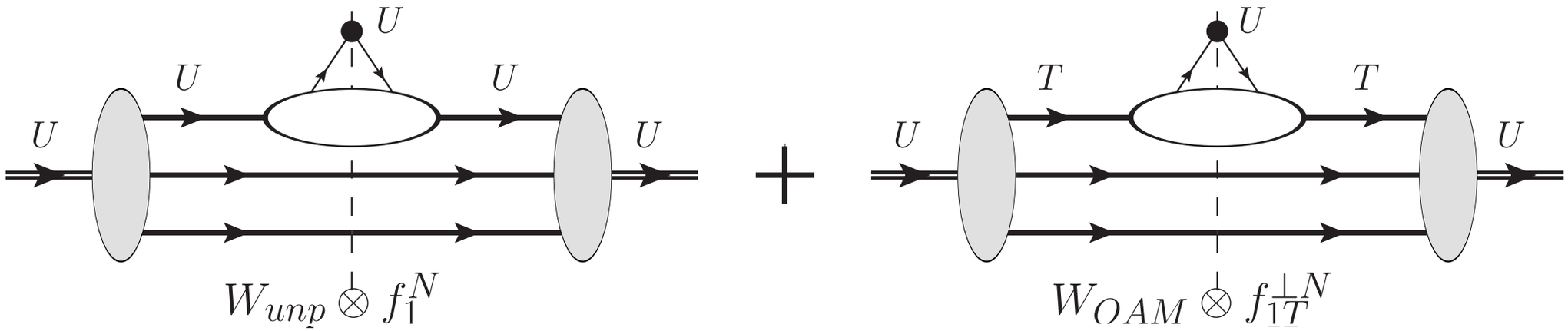
Unpolarized quarks, azimuthally symmetric

Boer-Mulders Distribution: Transversely polarized quarks, azimuthally antisymmetric

Unpolarized Quark Distribution f_1^A

$f_1^A =$



$$f_1^A(x, k_T) = \frac{2A}{(2\pi)^5} \int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\vec{k}_\perp - \vec{k}'_\perp - \hat{x}\vec{p}_\perp)} S^{[\infty^-, b^-]}_{(r_T, b_T)}$$

$$\times \left(W_{unp}(p,b)\, f_1^N(\hat{x}, k'_T) - \frac{P^+ b^-}{A m_N^2} (\vec{p}_\perp \cdot \vec{k}'_\perp)\, W_{OAM}(p,b)\, f_{1T}^{\perp N}(\hat{x}, k'_T) \right)$$

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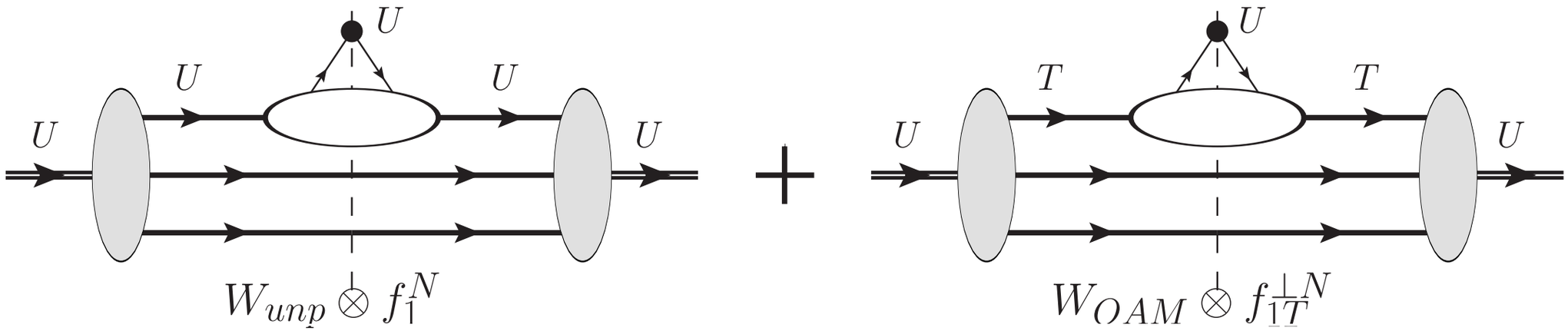

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Unpolarized Channel: $f_1^N \to f_1^A$

- Trivial density distribution + momentum broadening

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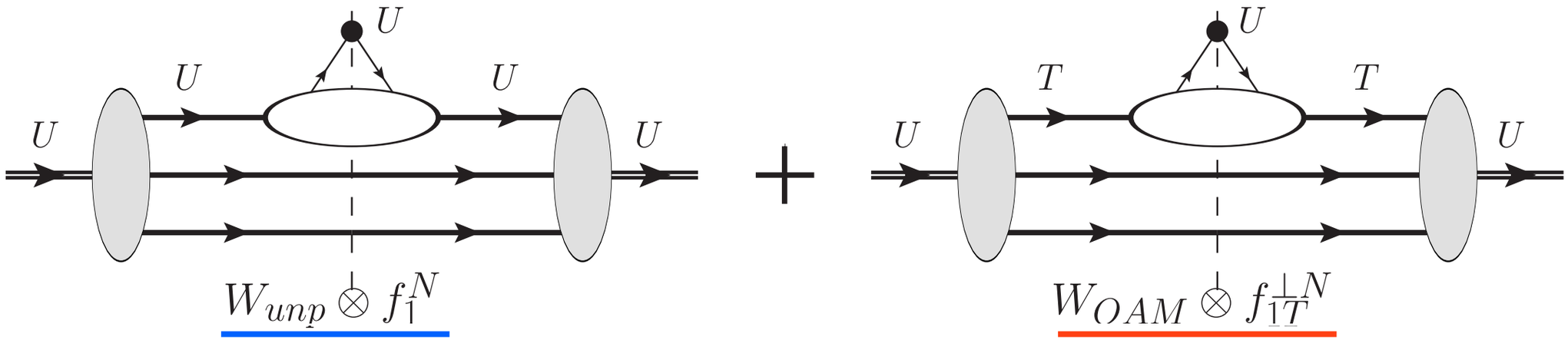

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Unpolarized Channel: $f_1^N \to f_1^A$

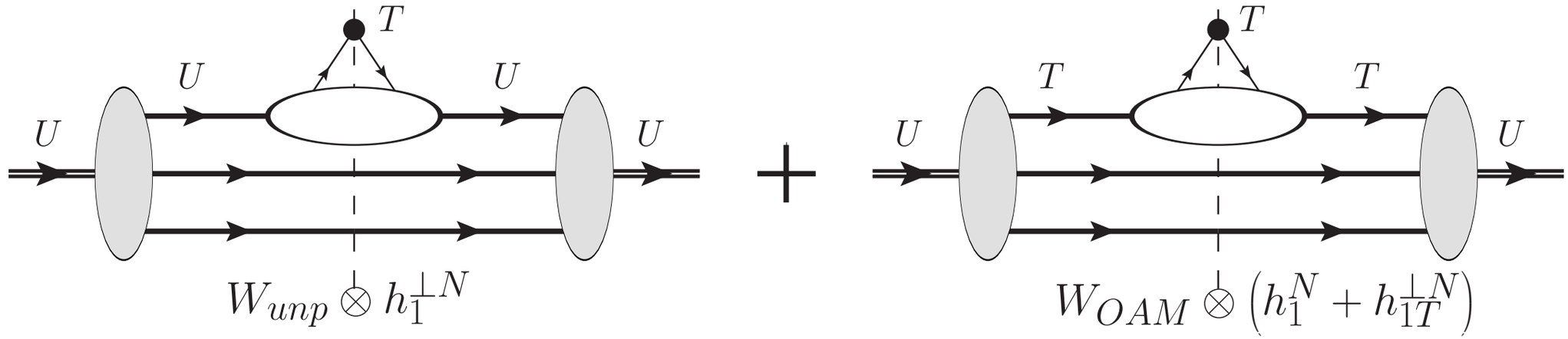
- Trivial density distribution + momentum broadening

OAM Channel: $f_{1T}^{\perp N} \to f_1^A$

- "Dipole modulation": Sivers function + momentum broadening

Boer-Mulders Distribution $h_1^{\perp A}$

$h_1^{\perp A} =$



$$h_1^{\perp A}(x,k_T) = \frac{2A}{(2\pi)^5}\frac{Am_N}{k_T^2}\int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\vec{k}_\perp - \vec{k}'_\perp - \hat{x}\vec{p}_\perp)\cdot\vec{r}_\perp} S^{[\infty^-,b^-]}_{(r_T,b_T)}$$

$$\times\left(\frac{\vec{k}_\perp\cdot\vec{k}'_\perp}{m_N} W_{unp}(p,b)\, h_1^{\perp N}(\hat{x},k'_T) - \frac{P^+b^-}{Am_N}(\vec{p}_\perp\cdot\vec{k}_\perp)\, W_{OAM}(p,b)\, h_1^N(\hat{x},k'_T)\right.$$

$$\left. - \frac{P^+b^-}{Am_N}\left(\frac{(\vec{p}_\perp\times\vec{k}'_\perp)(\vec{k}_\perp\times\vec{k}'_\perp)}{m_N^2} - \frac{k_T'^2(\vec{p}_\perp\cdot\vec{k}_\perp)}{2m_N^2}\right) W_{OAM}(p,b)\, h_{1T}^{\perp N}(\hat{x},k'_T)\right)$$

Boer-Mulders Distribution $h_1^{\perp A}$

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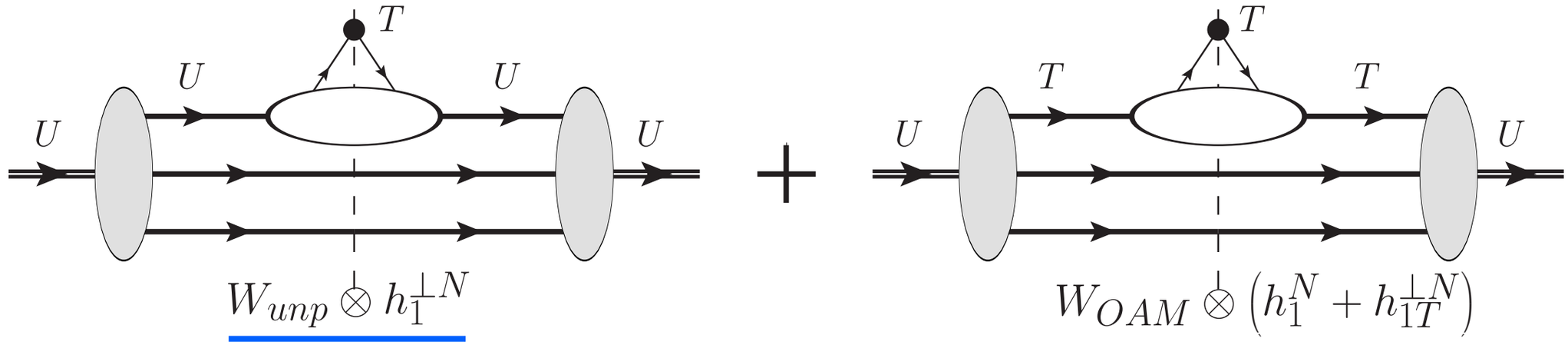

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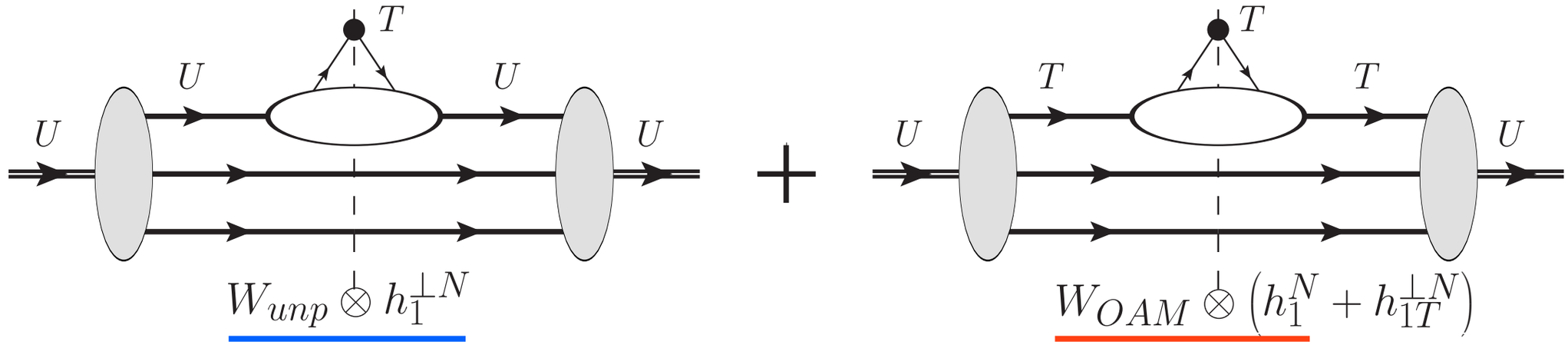
Unpolarized Channel: $h_1^{\perp N} \to h_1^{\perp A}$

- Trivial density distribution + momentum broadening

Boer-Mulders Distribution $h_1^{\perp A}$

$h_1^{\perp A} =$



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Unpolarized Channel: $h_1^{\perp N} \to h_1^{\perp A}$

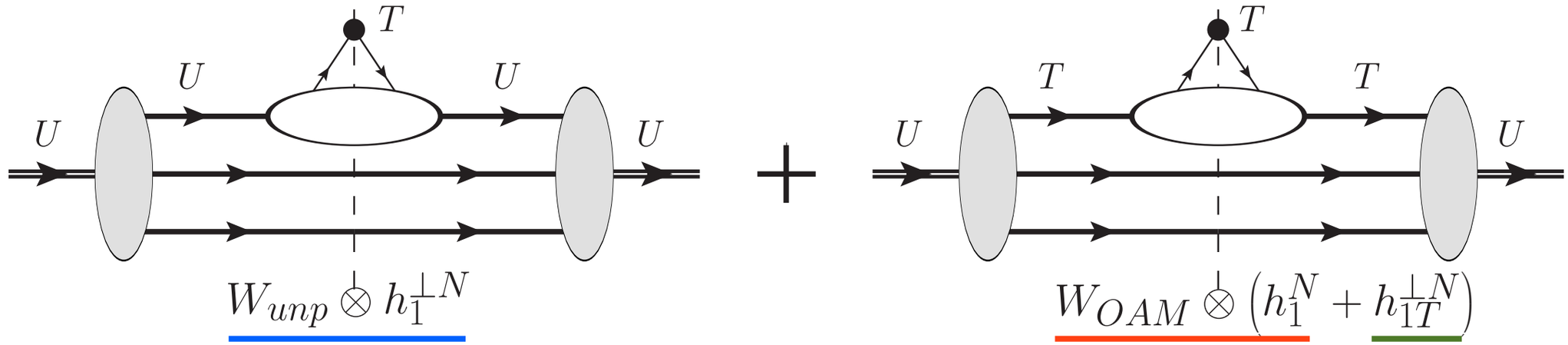
- Trivial density distribution + momentum broadening

OAM Channel: h_1^N , $h_{1T}^{\perp N} \to h_1^{\perp A}$

- "Dipole modulation": Transversity + broadening

Boer-Mulders Distribution $h_1^{\perp A}$

$h_1^{\perp A} =$



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Unpolarized Channel: $h_1^{\perp N} \to h_1^{\perp A}$

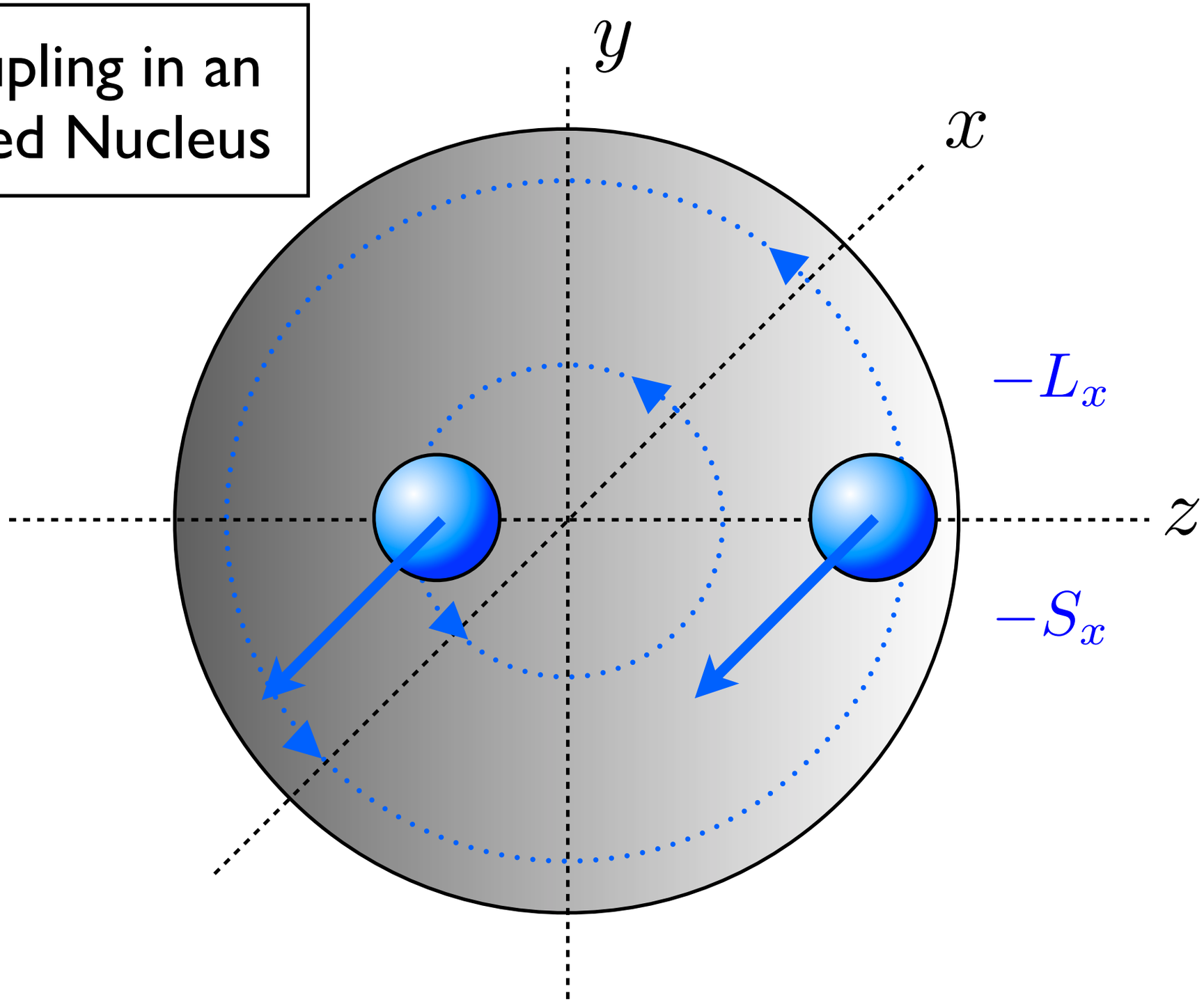
- Trivial density distribution + momentum broadening

OAM Channel: $h_1^N\,,\, h_{1T}^{\perp N} \to h_1^{\perp A}$

- "Dipole modulation": Transversity + broadening
- "Quadrupole modulation": Pretzelosity + broadening

A Physical Illustration: $W_{OAM} + h_1^N \to h_1^{\perp A}$

$\vec{L} \cdot \vec{S}$ Coupling in an Unpolarized Nucleus



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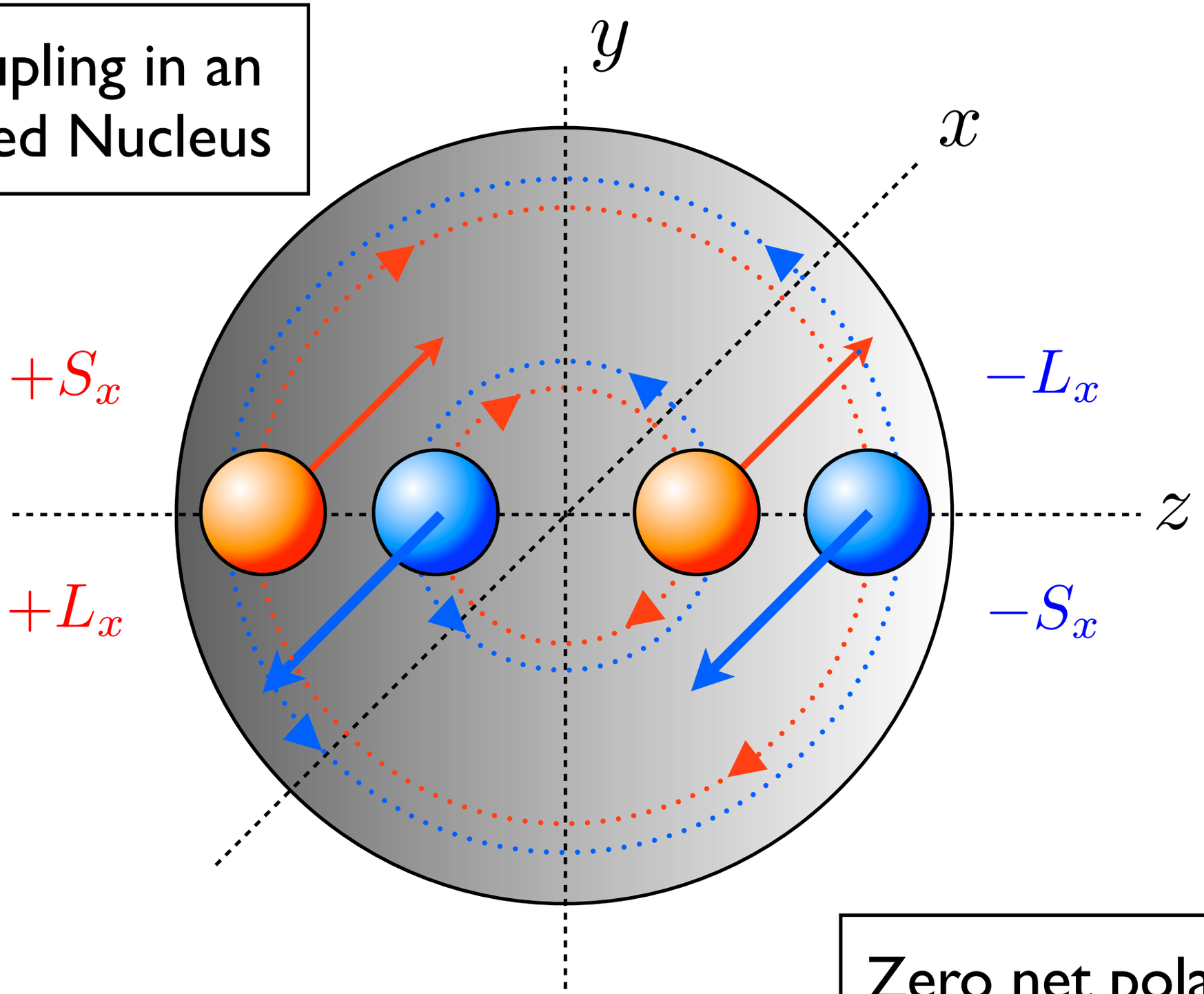
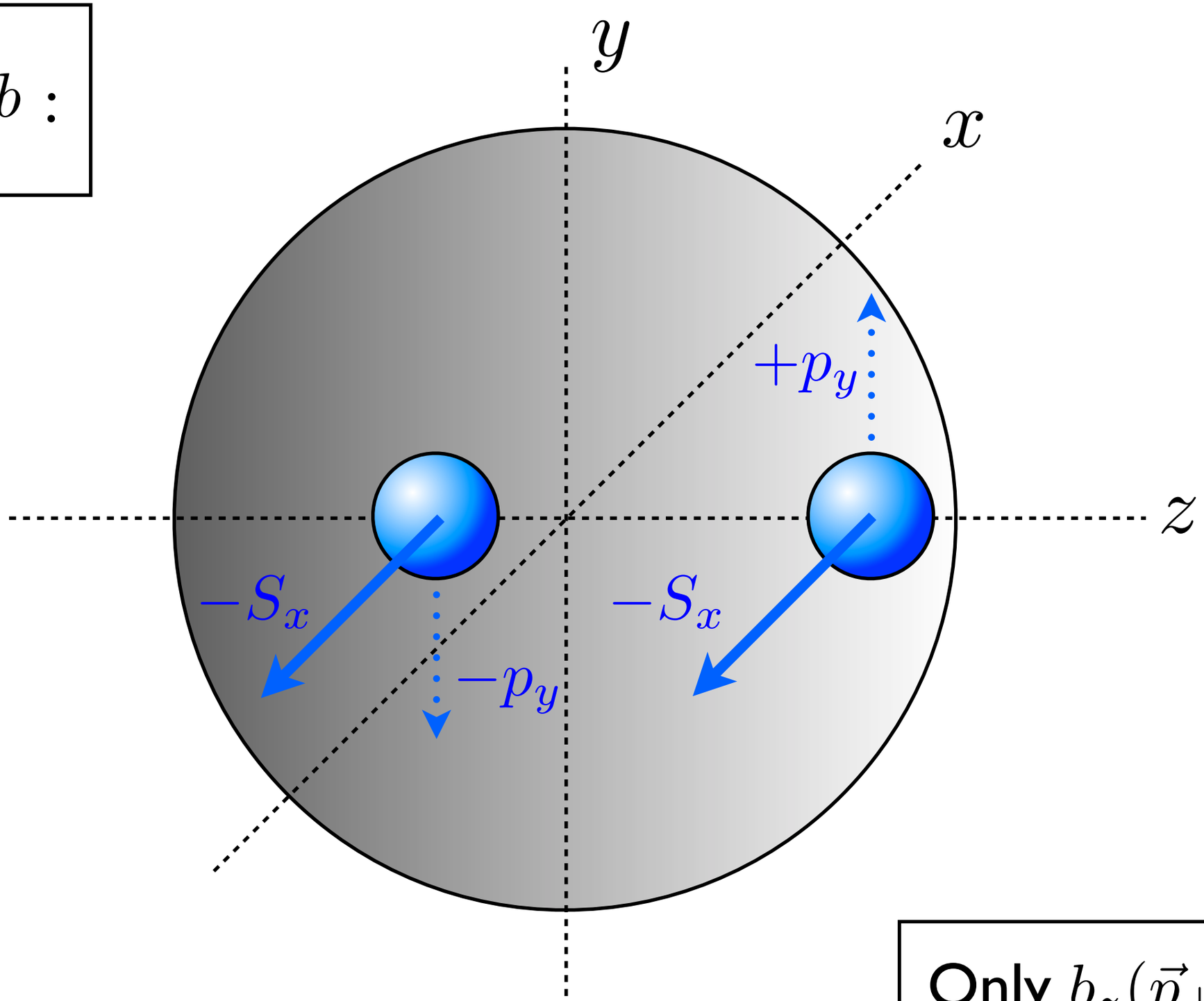


Zero net polarization.

A Physical Illustration: $W_{OAM} + h_1^N \to h_1^{\perp A}$

After $\int d^2b$:



Only $b_z(\vec{p}_\perp \times \vec{S}_\perp)$

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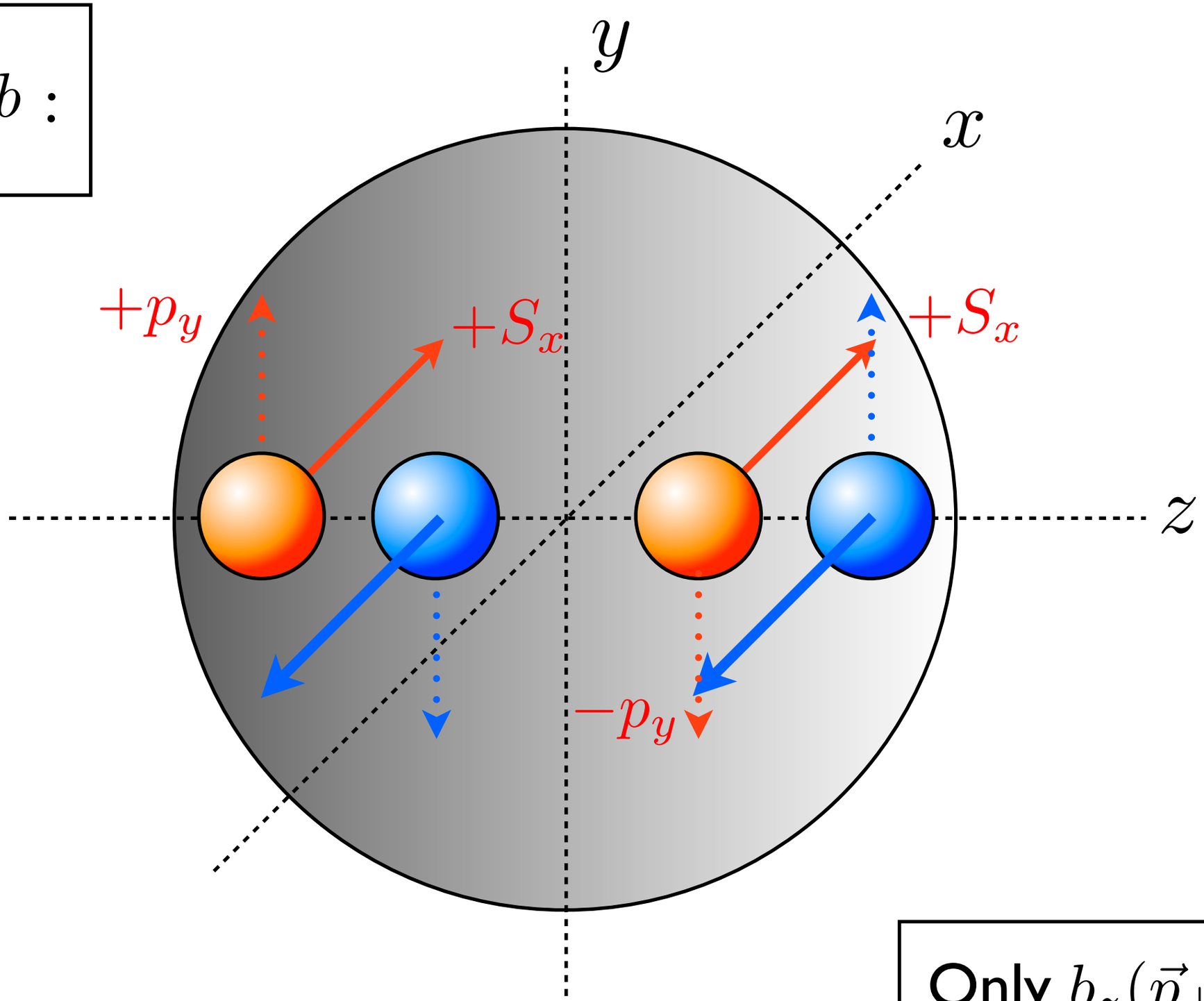

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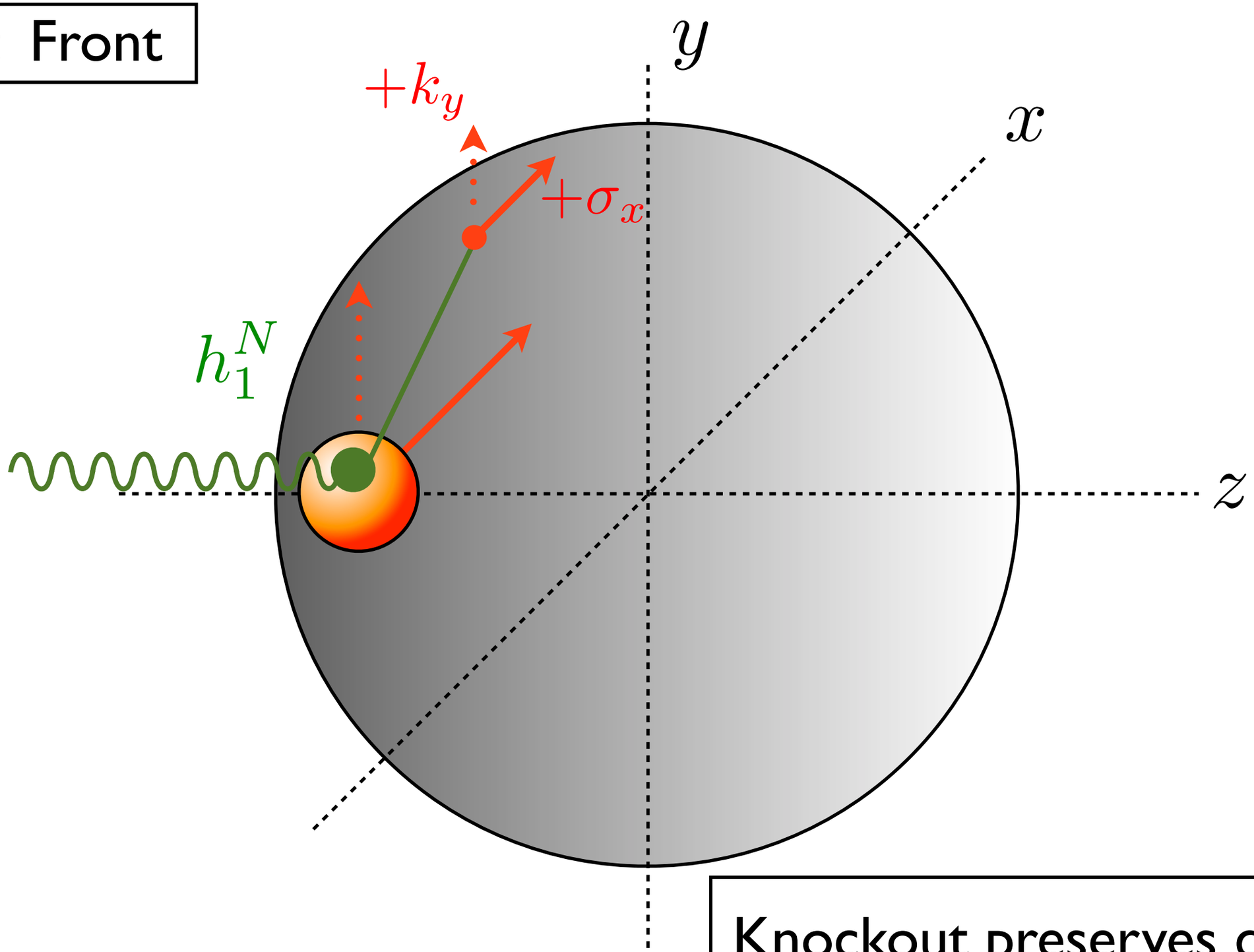
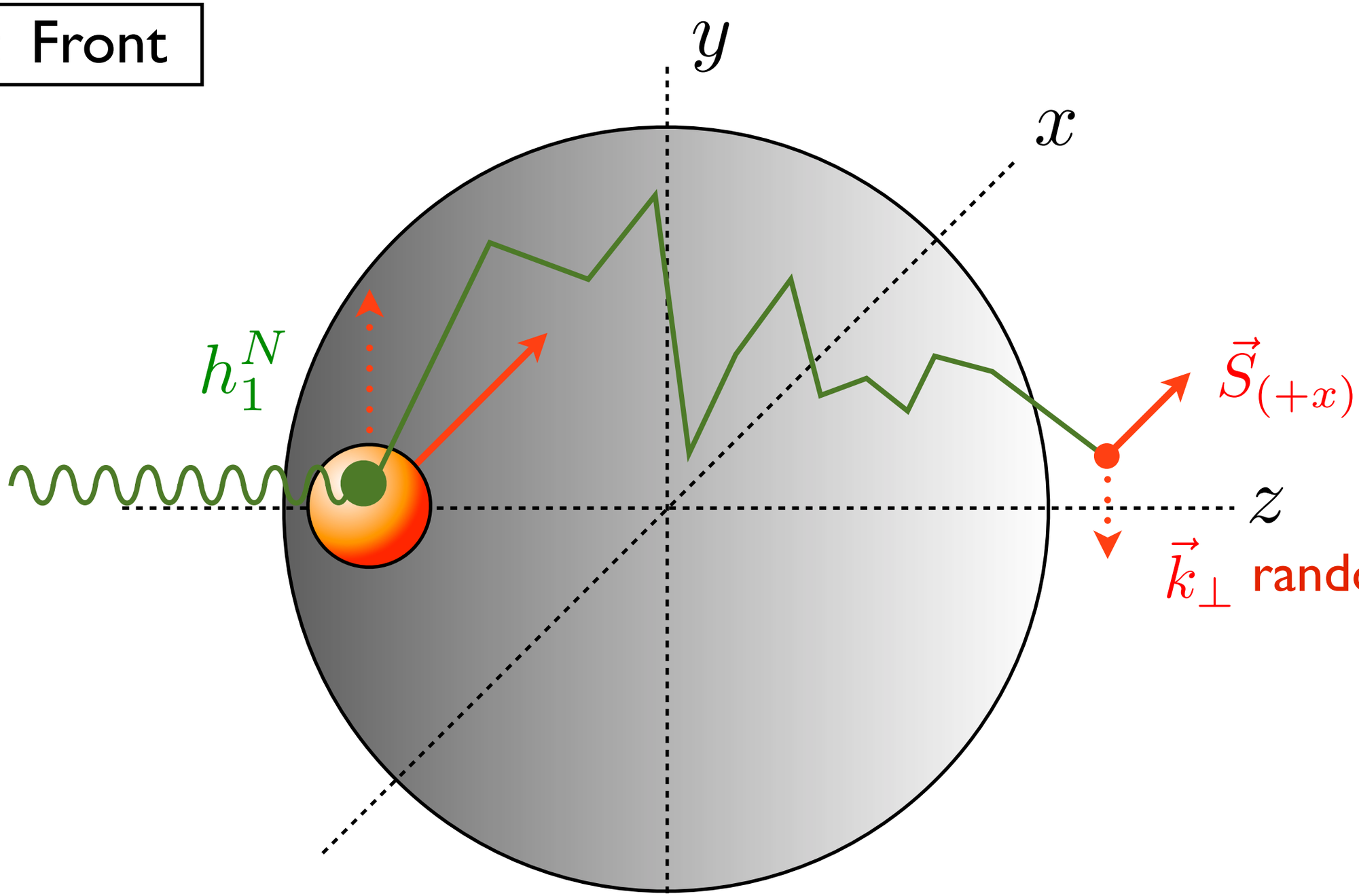
SIDIS: Front



Knockout preserves direction.

A Physical Illustration: $W_{OAM} + h_1^N \to h_1^{\perp A}$

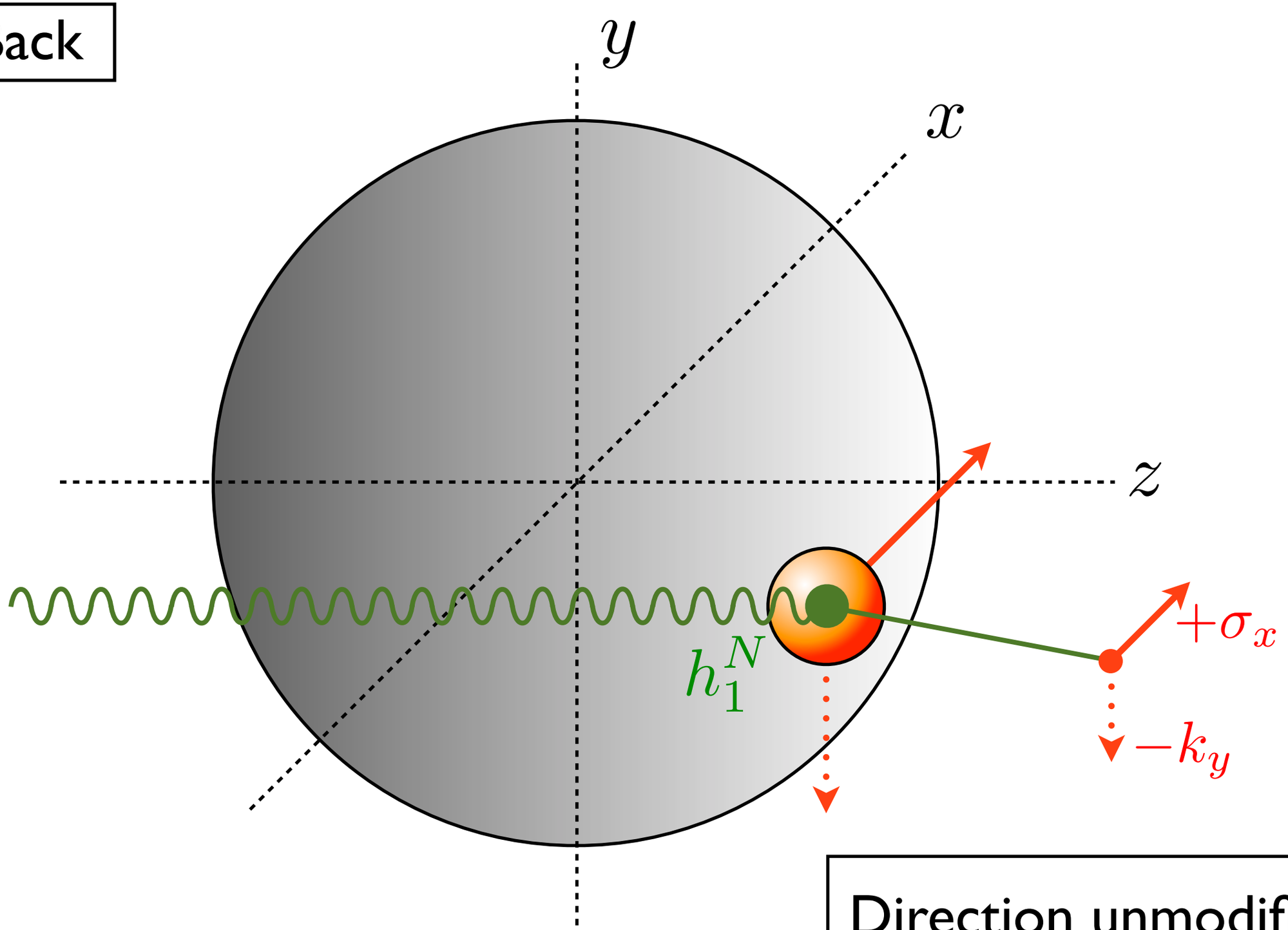
SIDIS: Front



Knockout preserves direction.
But recattering randomizes it.

A Physical Illustration: $W_{OAM} + h_1^N \to h_1^{\perp A}$

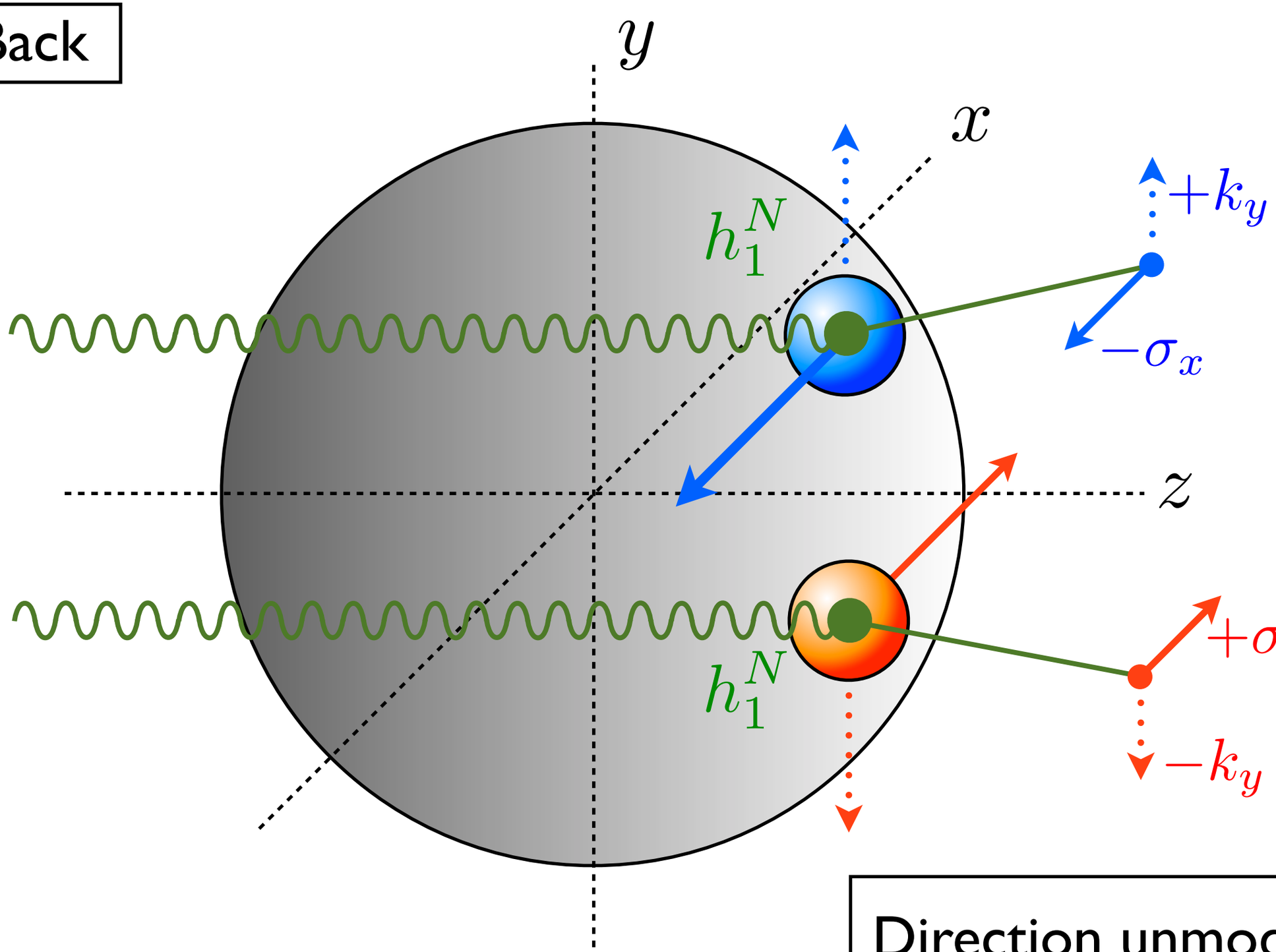
SIDIS: Back



Direction unmodified.

A Physical Illustration: $W_{OAM} + h_1^N \to h_1^{\perp A}$

SIDIS: Back



Direction unmodified.

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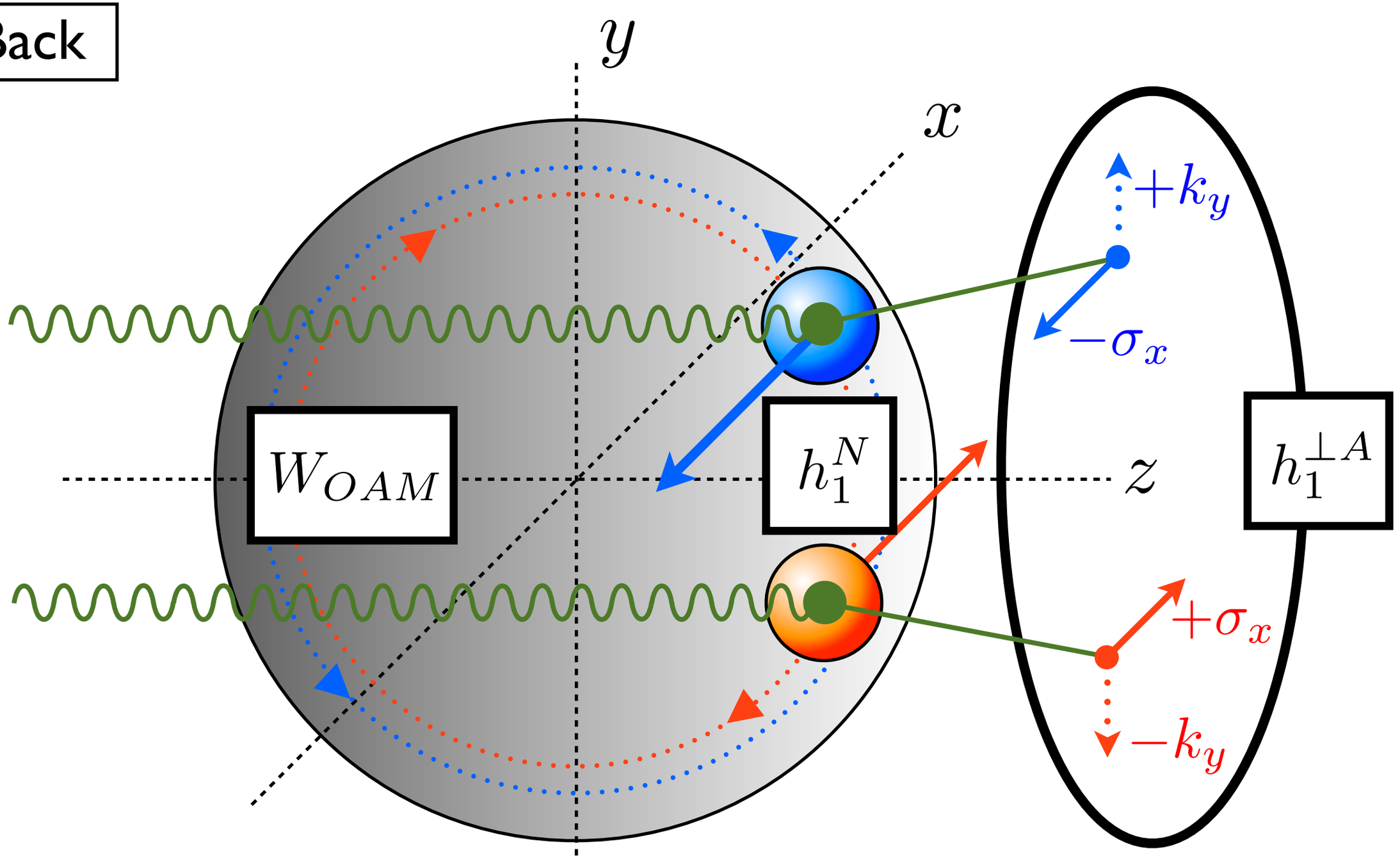
SIDIS: Back



The Result: $W_{OAM} + h_1^N \to h_1^{\perp A}$

TMD Mixing

Spin-Orbit Coupling
W_{OAM}

$+$

Momentum Broadening
$S^{[\infty^-,b^-]}_{(r_T,b_T)}$

$\rightarrow$

TMD Mixing

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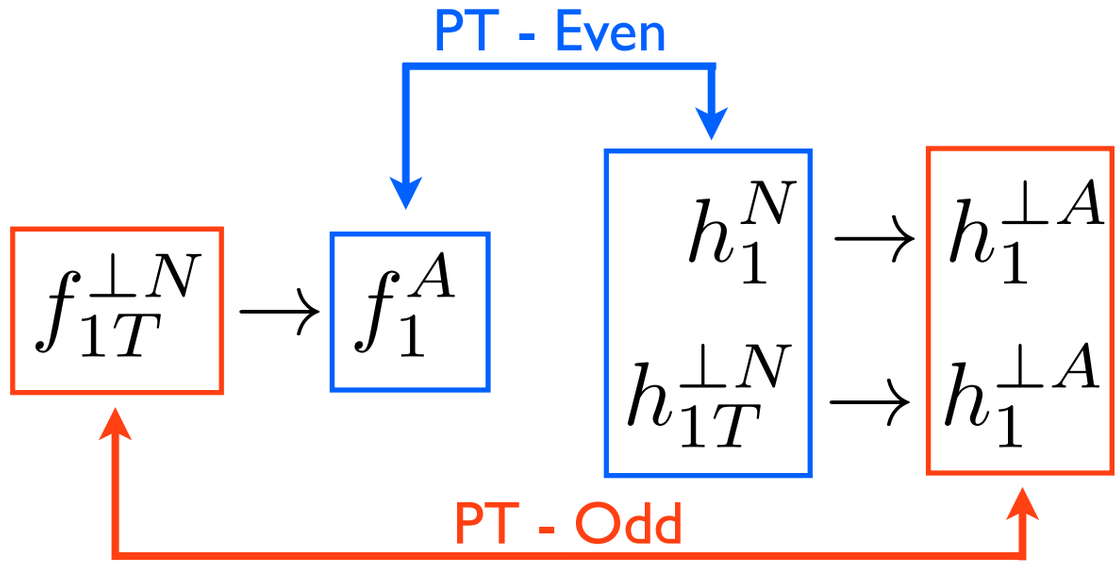
Mixing occurs between the PT-even and PT-odd sectors:



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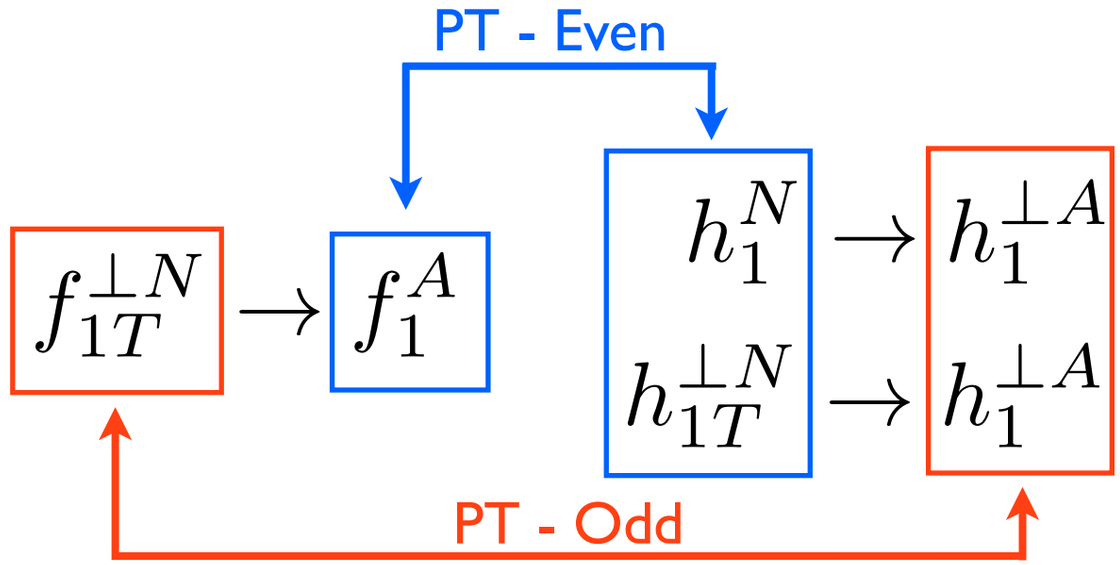

Multiple rescattering is essential to break front / back symmetry

- b_z dependence provides a PT-reversing factor
- OAM is not enough: mixing vanishes without rescattering.

Implications for an EIC

(1.)

Deviation from simple broadening?

$$f_1^A \Leftrightarrow f_1^N$$

Presence of OAM!

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If the nucleonic TMD's are known...	...and the nuclear TMD's are measured...	...can directly extract OAM
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(3.) This provides a prediction for additional mixing in other sectors!

$$h_1^{\perp A} \sim h_1^{\perp N} \otimes W_{unp} + (h_1^N + h_{1T}^{\perp N}) \otimes W_{OAM}$$

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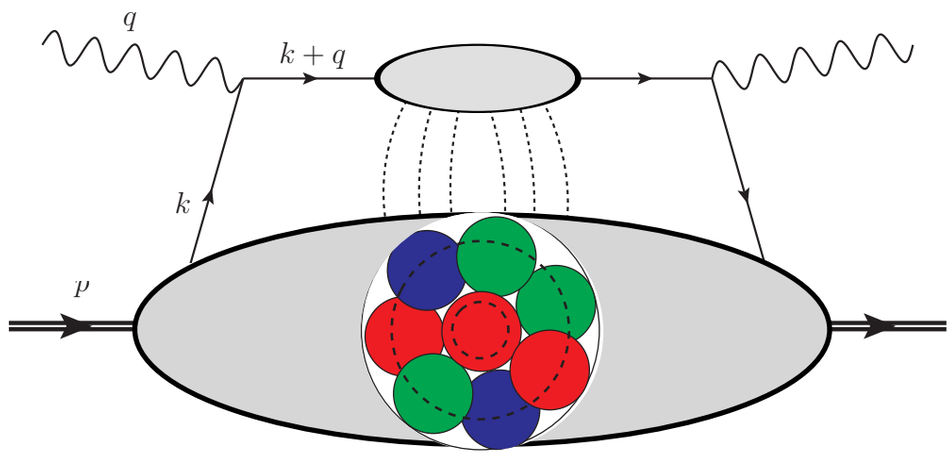
  - Quantum evolution corrections
    - Large-x Q^2 evolution (CSS)
    - Small-x evolution: polarized and unpolarized.

Part II

4:24 pm, "Collective Behavior of Partons"

Summary

The gauge link which is essential for TMD structure represents very different physics at high density.



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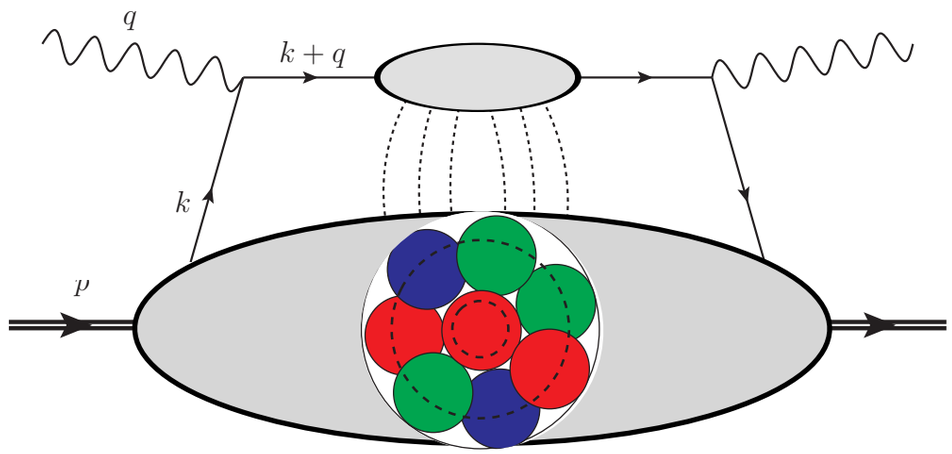
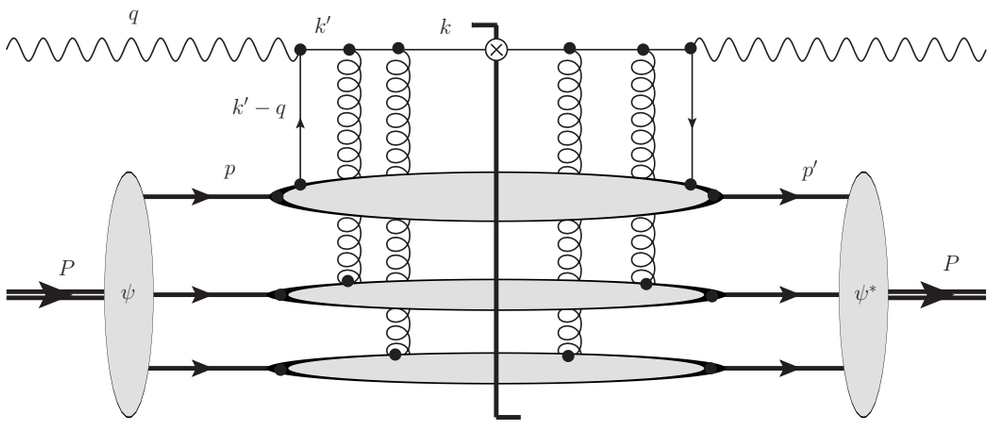


High-density effects can be re-summed, leading to a quasi-classical factorization of nuclear TMD's.



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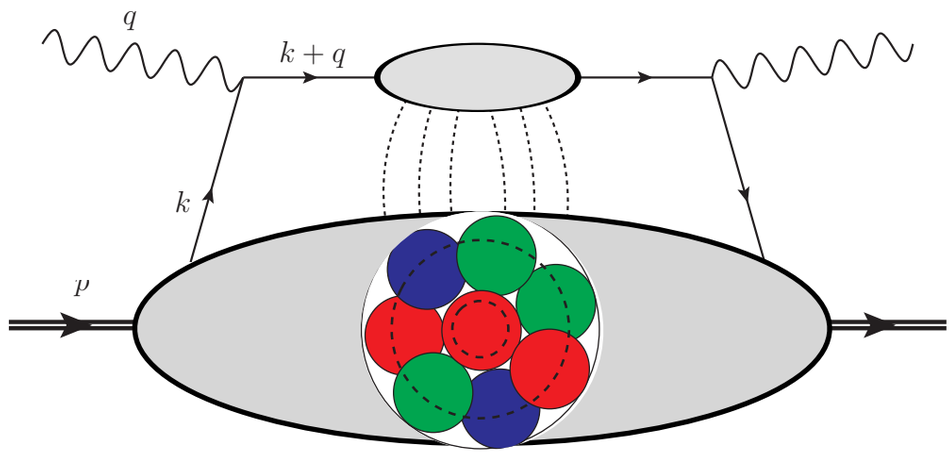

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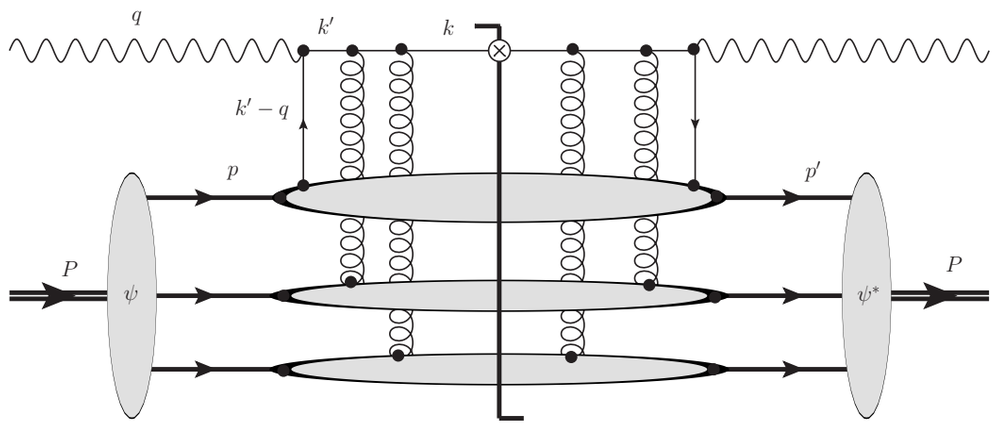

Spin-orbit coupling, together with multiple rescattering, leads to rich TMD mixing with predictive power.